18 FORECASTING

Learning Objectives

LO18–1 Understand how forecasting is essential to supply chain planning.

LO18–2 Evaluate demand using quantitative forecasting models.

LO18–3 Apply qualitative techniques to forecast demand.

LO18–4 Apply collaborative techniques to forecast demand.

FROM BEAN TO CUP: STARBUCKS GLOBAL SUPPLY CHAIN CHALLENGE

Starbucks Corporation is the largest coffeehouse company in the world with over 17,000 stores in more than 50 countries. The company serves some 50 million customers each week.

Forecasting demand for a Starbucks is an amazing challenge. The product line goes well beyond drip-brewed coffee sold on demand in the stores. It includes espresso-based hot drinks, other hot and cold drinks, coffee beans, salads, hot and cold sandwiches and panini, pastries, snacks, and items such as mugs and tumblers. Through its entertainment division and the Hear Music brand, the company also markets books, music, and videos. Many of the company's products are seasonal or specific to the locality of the store. Starbucks-branded ice cream and coffee are also offered at grocery stores around the world.

Starbucks Cafe in Bur Juman Center Shopping Mall, Dubai, United Arab Emirates.

The creation of a single, global logistics system was important for Starbucks because of its far-flung supply chain. The company generally brings coffee beans from Latin America, Africa, and Asia to the United States and Europe in ocean containers. From the port of entry, the "green" (unroasted) beans are trucked to six storage sites, either at a roasting plant or nearby. After the beans are roasted and packaged, the finished product is trucked to regional distribution centers, which range from 200,000 to 300,000 square feet in size. Starbucks runs five regional distribution centers (DCs) in the United States. It has two DCs in Europe and two more in Asia. Coffee, however, is only one of the many products held at these warehouses. They also handle other items required by Starbucks retail outlets, everything from furniture to cappuccino mix.

In the Analytics Exercise at the end of the chapter, we consider the challenging demand forecasting problem that Starbucks must solve to be able to successfully run this complex supply chain.

Source: Based on James A. Cooke, "From bean to cup: How Starbucks transformed its supply chain," *CSCUPs Supply Chain Quarterly,* Fourth Quarter, 2010.

LO18–1

Understand how forecasting is essential to supply chain planning.

FORECASTING IN OPERATIONS AND SUPPLY CHAIN MANAGEMENT

Forecasts are vital to every business organization and for every significant management decision. Forecasting is the basis of corporate planning and control. In the functional areas of finance and accounting, forecasts provide the basis for budgetary planning and cost control. Marketing relies on sales forecasting to plan new products, compensate sales personnel, and make other key decisions. Production and operations personnel use forecasts to make periodic decisions involving supplier selection, process selection, capacity planning, and facility layout, as well as for continual decisions about purchasing, production planning, scheduling, and inventory.

In considering what forecasting approach to use, it is important to consider the purpose of the forecast. Some forecasts are for very high-level demand analysis. What do we expect the demand to be for a group of products over the next year, for example? Some forecasts are used to help set the strategy of how, in an aggregate sense, we will meet demand. We will call these **strategic forecasts**. Relative to the material in the book, strategic forecasts are most appropriate when making decisions related to overall strategy (Chapter 2), capacity (Chapter 5), manufacturing process design (Chapter 7), service process design (Chapter 9), location and distribution design (Chapter 15), sourcing (Chapter 16), and in sales and operations planning (Chapter 19). These all involve medium and long-term decisions that relate to how demand will be met strategically.

Strategic forecasts
Medium and long-term forecasts that are used for decisions related to strategy and aggregate demand.

Forecasts are also needed to determine how a firm operates processes on a day-to-day basis. For example, when should the inventory for an item be replenished, or how much production should we schedule for an item next week? These are **tactical forecasts** where the goal is to estimate demand in the relatively short term, a few weeks or months. These forecasts are important to ensure that in the short term we are able to meet customer lead time expectations and other criteria related to the availability of our products and services.

Tactical forecasts
Short-term forecasts used for making day-to-day decisions related to meeting demand.

In Chapter 7, the concept of decoupling points is discussed. These are points within the supply chain where inventory is positioned to allow processes or entities in the supply chain to operate independently. For example, if a product is stocked at a retailer, the customer pulls the item from the shelf and the manufacturer never sees a customer order. Inventory acts as a buffer to separate the customer from the manufacturing process. Selection of decoupling points is a strategic decision that determines customer lead times and can greatly impact inventory investment. The closer this point is to the customer, the quicker the customer can be served. Typically, a trade-off is involved where quicker response to customer demand comes at the expense of greater inventory investment because finished goods inventory is more expensive than raw material inventory.

Strategy

Forecasting is needed at these decoupling points to set appropriate inventory levels for these buffers. The actual setting of these levels is the topic of Chapter 20, "Inventory Management," but an essential input into those decisions is a forecast of expected demand and the expected error associated with that demand. If, for example, we are able to forecast demand very accurately, then inventory levels can be set precisely to expected customer demand. On the other hand, if predicting short-term demand is difficult, then extra inventory to cover this uncertainty will be needed.

FORECASTING IS CRITICAL IN DETERMINING HOW MUCH INVENTORY TO KEEP TO MEET CUSTOMER NEEDS.

The same is true relative to service settings where inventory is not used to buffer demand. Here capacity availability relative to expected demand is the issue. If we can predict demand in a service setting very accurately, then tactically all we need to do is ensure that we have the appropriate capacity in the short term. When demand is not predictable, then excess capacity may be needed if servicing customers quickly is important.

Bear in mind that a perfect forecast is virtually impossible. Too many factors in the business environment cannot be predicted with certainty. Therefore, rather than search for the perfect forecast, it is far more important to establish the practice of continual review of forecasts and to learn to live with inaccurate forecasts. This is not to say that we should not try to improve the forecasting model or methodology or even to try to influence demand in a way that reduces demand uncertainty. When forecasting, a good strategy is to use two or three methods and look at them for the commonsense view. Will expected changes in the general economy affect the forecast? Are there changes in our customers' behaviors that will impact demand that are not being captured by our current approaches? In this chapter, we look at both *qualitative* techniques that use managerial judgment and also *quantitative* techniques that rely on mathematical models. It is our view that combining these techniques is essential to a good forecasting process that is appropriate to the decisions being made.

LO18–2

Evaluate demand using quantitative forecasting models.

QUANTITATIVE FORECASTING MODELS

Forecasting can be classified into four basic types: *qualitative, time series analysis, causal relationships*, and *simulation.* Qualitative techniques are covered later in the chapter. **Time series analysis**, the primary focus of this chapter, is based on the idea that data relating to past demand can be used to predict future demand. Past data may include several components, such as trend, seasonal, or cyclical influences, and are described in the following section. Causal forecasting, which we discuss using the linear regression technique, assumes that demand is related to some underlying factor or factors in the environment. Simulation models allow the forecaster to run through a range of assumptions about the condition of the forecast. In this chapter we focus on qualitative and time series techniques since these are most often used in supply chain planning and control.

Time series analysis
A forecast in which past demand data is used to predict future demand.

Components of Demand

In most cases, demand for products or services can be broken down into six components: average demand for the period, a trend, seasonal element, cyclical elements, random variation, and autocorrelation. Exhibit 18.1 illustrates a demand over a four-year period, showing the average, trend, and seasonal components and randomness around the smoothed demand curve.

Cyclical factors are more difficult to determine because the time span may be unknown or the cause of the cycle may not be considered. Cyclical influence on demand may come from such occurrences as political elections, war, economic conditions, or sociological pressures.

Random variations are caused by chance events. Statistically, when all the known causes for demand (average, trend, seasonal, cyclical, and autocorrelative) are subtracted from total demand, what remains is the unexplained portion of demand. If we cannot identify the cause of this remainder, it is assumed to be purely random chance.

Autocorrelation denotes the persistence of occurrence. More specifically, the value expected at any point is highly correlated with its own past values. In waiting line theory, the length of a waiting line is highly autocorrelated. That is, if a line is relatively long at one time, then shortly after that time, we would expect the line still to be long.

When demand is random, it may vary widely from one week to another. Where high autocorrelation exists, the rate of change in demand is not expected to change very much from one week to the next.

Trend lines are the usual starting point in developing a forecast. These trend lines are then adjusted for seasonal effects, cyclical elements, and any other expected events that may influence the final forecast. Exhibit 18.2 shows four of the most common types of trends. A linear trend is obviously a straight continuous relationship. An S-curve is typical of a product growth and maturity cycle. The most important point in the S-curve is where the trend changes from slow growth to fast growth or from fast to slow. An asymptotic trend starts with the highest demand growth at the beginning but then tapers off. Such a curve could happen when a firm enters an existing market with the objective of saturating and capturing a large share of the market. An exponential curve is common in products with explosive growth. The exponential trend suggests that sales will grow at an ever-increasing rate—an assumption that may not be safe to make.

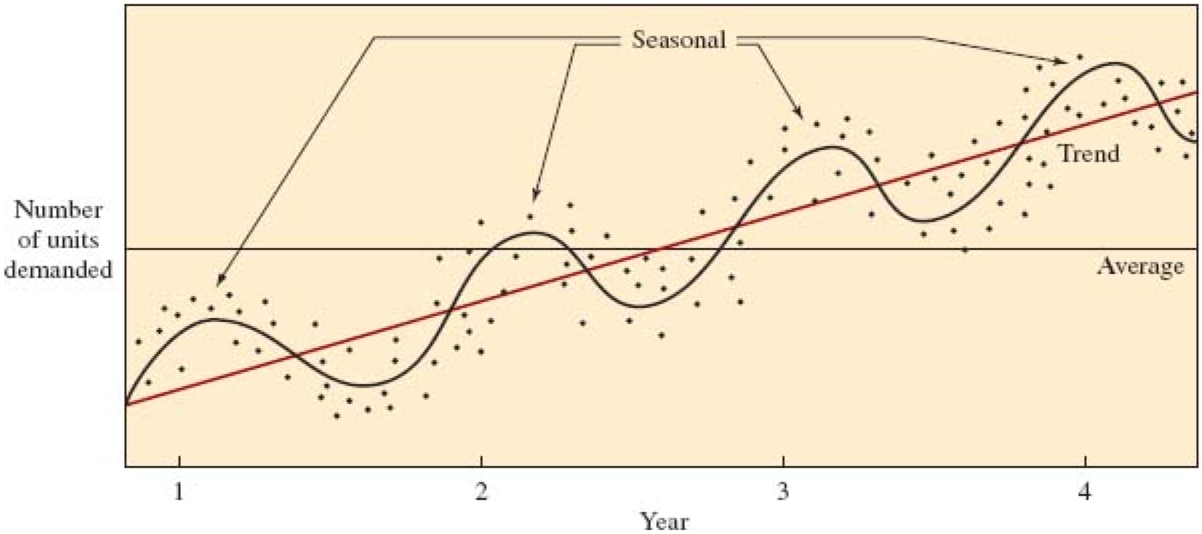
KEY IDEA

Remember that we are illustrating long-term patterns so they should be considered when making long-term forecasts. When making short-term forecasts, these patterns are often not so strong.

A widely used forecasting method plots data and then searches for the curve pattern (such as linear, S-curve, asymptotic, or exponential) that fits best. The attractiveness of this method is that because the mathematics for the curve are known, solving for values for future time periods is easy.

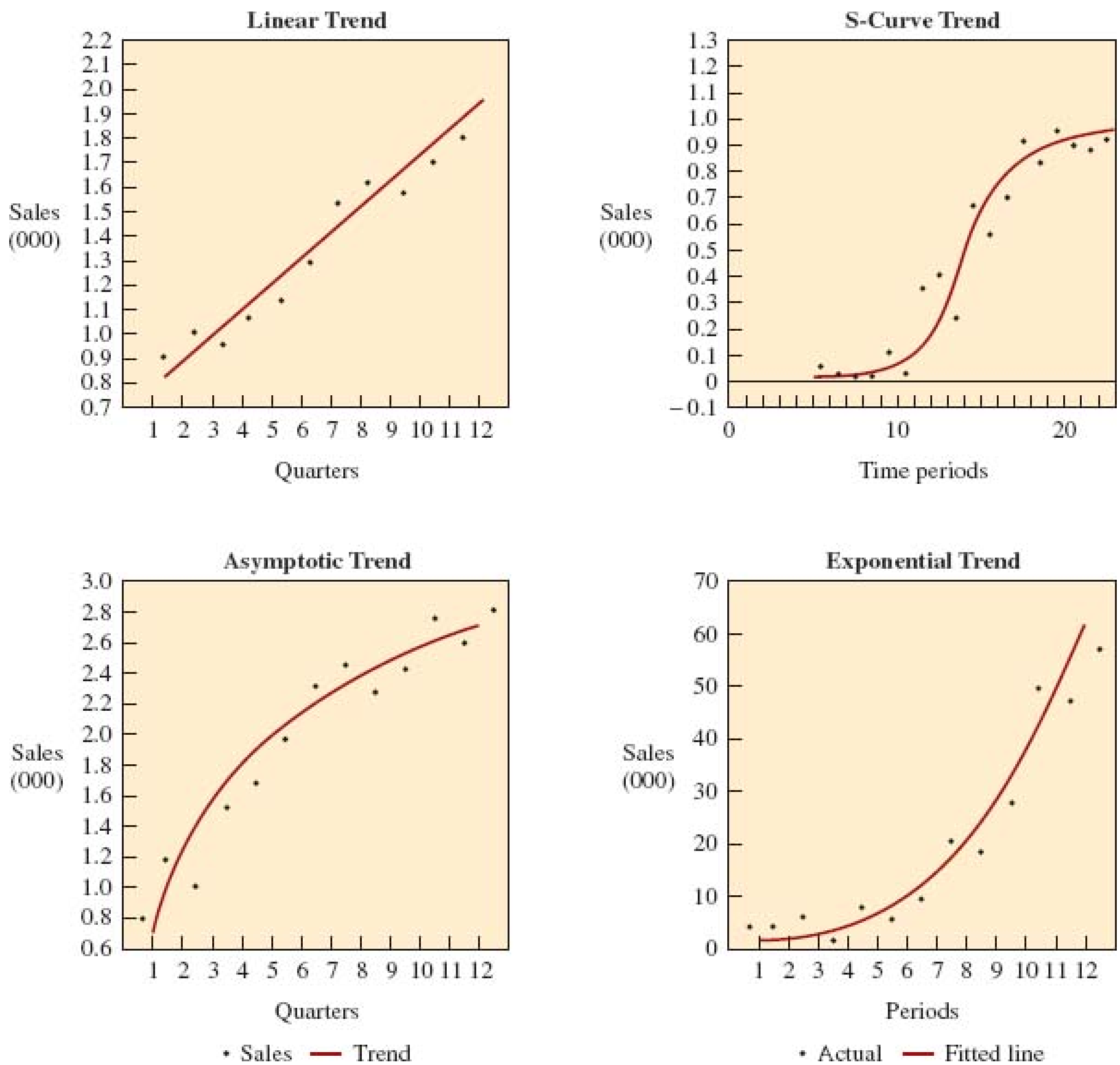
Sometimes our data do not seem to fit any standard curve. This may be due to several causes essentially beating the data from several directions at the same time. For these cases, a simplistic but often effective forecast can be obtained by simply plotting data.

exhibit 18.1 Historical Product Demand Consisting of a Growth Trend and Seasonal Demand



For the Excel template, visit www.mhhe.com/jacobs14e.

exhibit 18.2 Common Types of Trends



Time Series Analysis

Time series forecasting models try to predict the future based on past data. For example, sales figures collected for the past six weeks can be used to forecast sales for the seventh week. Quarterly sales figures collected for the past several years can be used to forecast future quarters. Even though both examples contain sales, different forecasting time series models would likely be used.

Exhibit 18.3 shows the time series models discussed in the chapter and some of their characteristics. Terms such as *short, medium,* and *long* are relative to the context in which they are used. However, in business forecasting *short term* usually refers to under three months; *medium term,* three months to two years; and *long term,* greater than two years. We would generally use short-term forecasts for tactical decisions such as replenishing inventory or scheduling employees in the near term, and medium-term forecasts for planning a strategy for meeting demand over the next six months to a year and a half. In general, the short-term models compensate for random variation and adjust for short-term changes (such as consumers' responses to a new product). They are especially good for measuring the current variability in demand, which is useful for setting safety stock levels or estimating peak loads in a service setting. Medium-term forecasts are useful for capturing seasonal effects, and long-term models detect general trends and are especially useful in identifying major turning points.

Which forecasting model a firm should choose depends on:

1. Time horizon to forecast
2. Data availability
3. Accuracy required

exhibit 18.3 A Guide to Selecting an Appropriate Forecasting Method

Forecasting Method	Amount of Historical Data	Data Pattern	Forecast Horizon
Simple moving average	6 to 12 months; weekly data are often used	Stationary only (i.e., no trend or seasonality)	Short
Weighted moving average and simple exponential smoothing	5 to 10 observations needed to start	Stationary only	Short
Exponential smoothing with trend	5 to 10 observations needed to start	Stationary and trend	Short
Linear regression	10 to 20 observations	Stationary, trend, and seasonality	Short to medium
Trend and seasonal models	2 to 3 observations per season	Stationary, trend, and seasonality	Short to medium

4. Size of forecasting budget
5. Availability of qualified personnel

To view a tutorial on Forecasting, visit www.mhhe.com/jacobs14e_tutorial_ch18.

In selecting a forecasting model, there are other issues such as the firm's degree of flexibility. (The greater the ability to react quickly to changes, the less accurate the forecast needs to be.) Another item is the consequence of a bad forecast. If a large capital investment decision is to be based on a forecast, it should be a good forecast.

Simple Moving Average When demand for a product is neither growing nor declining rapidly, and if it does not have seasonal characteristics, a **moving average** can be useful in removing the random fluctuations for forecasting. The idea here is to simply calculate the average demand over the most recent periods. Each time a new forecast is made, the oldest period is discarded in the average and the newest period included. Thus, if we want to forecast June with a five-month moving average, we can take the average of January, February, March, April, and May. When June passes, the forecast for July would be the average of February, March, April, May, and June. An example using weekly demand is shown in Exhibit 18.4. Here, 3-week and 9-week moving average forecasts are calculated. Notice how the forecast is shown in the period following the data used. The 3-week moving average for week 4 uses actual demand from weeks 1, 2, and 3.

Moving Average
A forecast based on average past demand.

Selecting the period length should be dependent on how the forecast is going to be used. For example, in the case of a medium-term forecast of demand for planning a budget, monthly time periods might be more appropriate, whereas, if the forecast were being used for a short-term decision related to replenishing inventory, a weekly forecast might be more appropriate. Although it is important to select the best period for the moving average, the number of periods to use in the forecast can also have a major impact on the accuracy of the forecast. As the moving average period becomes shorter, and fewer periods are used, and there is more oscillation, there is a closer following of the trend. Conversely, a longer time span gives a smoother response, but lags the trend.

The formula for a simple moving average is

$$F_t = \frac{A_{t-1} + A_{t-2} + A_{t-3} + \cdots + A_{t-n}}{n} \qquad \textbf{[18.1]}$$

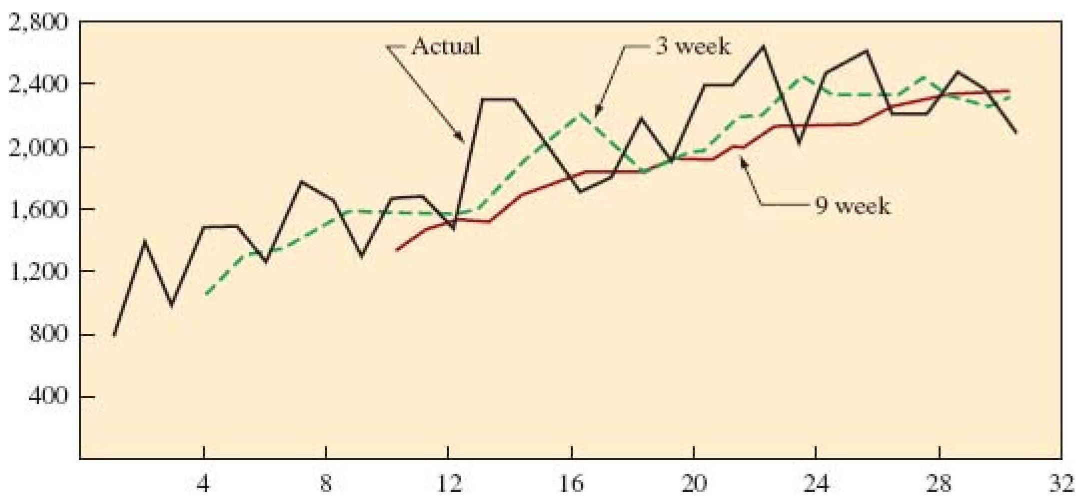
where

Ft = Forecast for the coming period
n = Number of periods to be averaged
At_{21} = Actual occurrence in the past period
A_{t-2}, A_{t-3}, and A_{t-n} = Actual occurrences two periods ago, three periods ago, and so on, up to n periods ago

exhibit 18.4 Forecast Demand Based on a Three- and a Nine-Week Simple Moving Average

Week	Demand	3 Week	9 Week	Week	Demand	3 Week	9 Week
1	800			16	1,700	2,200	1,811
2	1,400			17	1,800	2,000	1,800
3	1,000			18	2,200	1,833	1,811
4	1,500	1,067		19	1,900	1,900	1,911
5	1,500	1,300		20	2,400	1,967	1,933
6	1,300	1,333		21	2,400	2,167	2,011
7	1,800	1,433		22	2,600	2,233	2,111
8	1,700	1,533		23	2,000	2,467	2,144
9	1,300	1,600		24	2,500	2,333	2,111
10	1,700	1,600	1,367	25	2,600	2,367	2,167
11	1,700	1,567	1,467	26	2,200	2,367	2,267
12	1,500	1,567	1,500	27	2,200	2,433	2,311
13	2,300	1,633	1,556	28	2,500	2,333	2,311
14	2,300	1,833	1,644	29	2,400	2,300	2,378
15	2,000	2,033	1,733	30	2,100	2,367	2,378

excel
For the Excel template, visit www.mhhe.com/jacobs14e.



Analytics

A plot of the data in Exhibit 18.4 shows the effects of using different numbers of periods in the moving average. We see that the growth trend levels off at about the 23rd week. The three-week moving average responds better in following this change than the nine-week average, although overall, the nine-week average is smoother.

The main disadvantage in calculating a moving average is that all individual elements must be carried as data because a new forecast period involves adding new data and dropping the earliest data. For a three- or six-period moving average, this is not too severe. But plotting a 60-day moving average for the usage of each of 100,000 items in inventory would involve a significant amount of data.

Weighted Moving Average Whereas the simple moving average assigns equal importance to each component of the moving average database, a **weighted moving average** allows any weights to be placed on each element, provided, of course, that the sum of all weights equals 1. For example, a department store may find that, in a four-month period, the best forecast is derived by using 40 percent of the actual sales for the most recent month, 30 percent of two months ago, 20 percent of three months ago, and 10 percent of four months ago. If actual sales experience was

Weighted moving average
A forecast made with past data where more recent data is given more significance than older data.

Month 1	Month 2	Month 3	Month 4	Month 5
100	90	105	95	?

the forecast for month 5 would be

$$\begin{aligned} F_5 &= 0.40(95) + 0.30(105) + 0.20(90) + 0.10(100) \\ &= 38 + 31.5 + 18 + 10 \\ &= 97.5 \end{aligned}$$

The formula for a weighted moving average is

$$F_t = W_1 A_{t-1} + W_2 A_{t-2} + \ldots + W_n A_{t-n} \quad \textbf{[18.2]}$$

where

W_1 = Weight to be given to the actual occurrence for the period $t - 1$
W_2 = Weight to be given to the actual occurrence for the period $t - 2$
W_n = Weight to be given to the actual occurrence for the period $t - n$
n = Total number of prior periods in the forecast

Although many periods may be ignored (that is, their weights are zero) and the weighting scheme may be in any order (for example, more distant data may have greater weights than more recent data), the sum of all the weights must equal 1.

$$\sum_{i=1}^{n} W_i = 1$$

Suppose sales for month 5 actually turned out to be 110. Then the forecast for month 6 would be

$$\begin{aligned} F_6 &= 0.40(110) + 0.30(95) + 0.20(105) + 0.10(90) \\ &= 44 + 28.5 + 21 + 9 \\ &= 102.5 \end{aligned}$$

Experience and trial and error are the simplest ways to choose weights. As a general rule, the most recent past is the most important indicator of what to expect in the future, and, therefore, it should get higher weighting. The past month's revenue or plant capacity, for example, would be a better estimate for the coming month than the revenue or plant capacity of several months ago.

However, if the data are seasonal, for example, weights should be established accordingly. Bathing suit sales in July of last year should be weighted more heavily than bathing suit sales in December (in the Northern Hemisphere).

The weighted moving average has a definite advantage over the simple moving average in being able to vary the effects of past data. However, it is more inconvenient and costly to use than the exponential smoothing method, which we examine next.

Exponential Smoothing In the previous methods of forecasting (simple and weighted moving averages), the major drawback is the need to continually carry a large amount of historical data. (This is also true for regression analysis techniques, which we soon will cover.) As each new piece of data is added in these methods, the oldest observation is dropped and the new forecast is calculated. In many applications (perhaps in most), the most recent occurrences are more indicative of the future than those in the more distant past. If this premise is valid—that the importance of data diminishes as the past becomes more distant—then **exponential smoothing** may be the most logical and easiest method to use.

Exponential smoothing
A time series forecasting technique using weights that decrease exponentially $(1 - \alpha)$ for each past period.

Exponential smoothing is the most used of all forecasting techniques. It is an integral part of virtually all computerized forecasting programs, and it is widely used in ordering inventory in retail firms, wholesale companies, and service agencies.

Exponential smoothing techniques have become well accepted for six major reasons:

1. Exponential models are surprisingly accurate.
2. Formulating an exponential model is relatively easy.
3. The user can understand how the model works.
4. Little computation is required to use the model.
5. Computer storage requirements are small because of the limited use of historical data.
6. Tests for accuracy as to how well the model is performing are easy to compute.

In the exponential smoothing method, only three pieces of data are needed to forecast the future: the most recent forecast, the actual demand that occurred for that forecast period, and a **smoothing constant alpha (β)**. This smoothing constant determines the level of smoothing and the speed of reaction to differences between forecasts and actual occurrences. The value for the constant is determined both by the nature of the product and by the manager's sense of what constitutes a good response rate. For example, if a firm produced a standard item with relatively stable demand, the reaction rate to differences between actual and forecast demand would tend to be small, perhaps just 5 or 10 percentage points. However, if the firm were experiencing growth, it would be desirable to have a higher reaction rate, perhaps 15 to 30 percentage points, to give greater importance to recent growth experience. The more rapid the growth, the higher the reaction rate should be. Sometimes users of the simple moving average switch to exponential smoothing but like to keep the forecasts about the same as the simple moving average. In this case, is approximated by $2 \div (n + 1)$, where n is the number of time periods in the corresponding simple moving average.

Smoothing constant alpha (α)
The parameter in the exponential smoothing equation that controls the speed of reaction to differences between forecasts and actual demand.

The equation for a single exponential smoothing forecast is simply

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) \qquad \textbf{[18.3]}$$

where

Ft	= The exponentially smoothed forecast for period t
F_{t-1}	= The exponentially smoothed forecast made for the prior period
A_{t-1}	= The actual demand in the prior period
α	= The desired response rate, or smoothing constant

This equation states that the new forecast is equal to the old forecast plus a portion of the error (the difference between the previous forecast and what actually occurred).

To demonstrate the method, assume that the long-run demand for the product under study is relatively stable and a smoothing constant (α) of 0.05 is considered appropriate. If the exponential smoothing method were used as a continuing policy, a forecast would have been made for last month. Assume that last month's forecast (Ft_{21}) was 1,050 units. If 1,000 actually were demanded, rather than 1,050, the forecast for this month would be

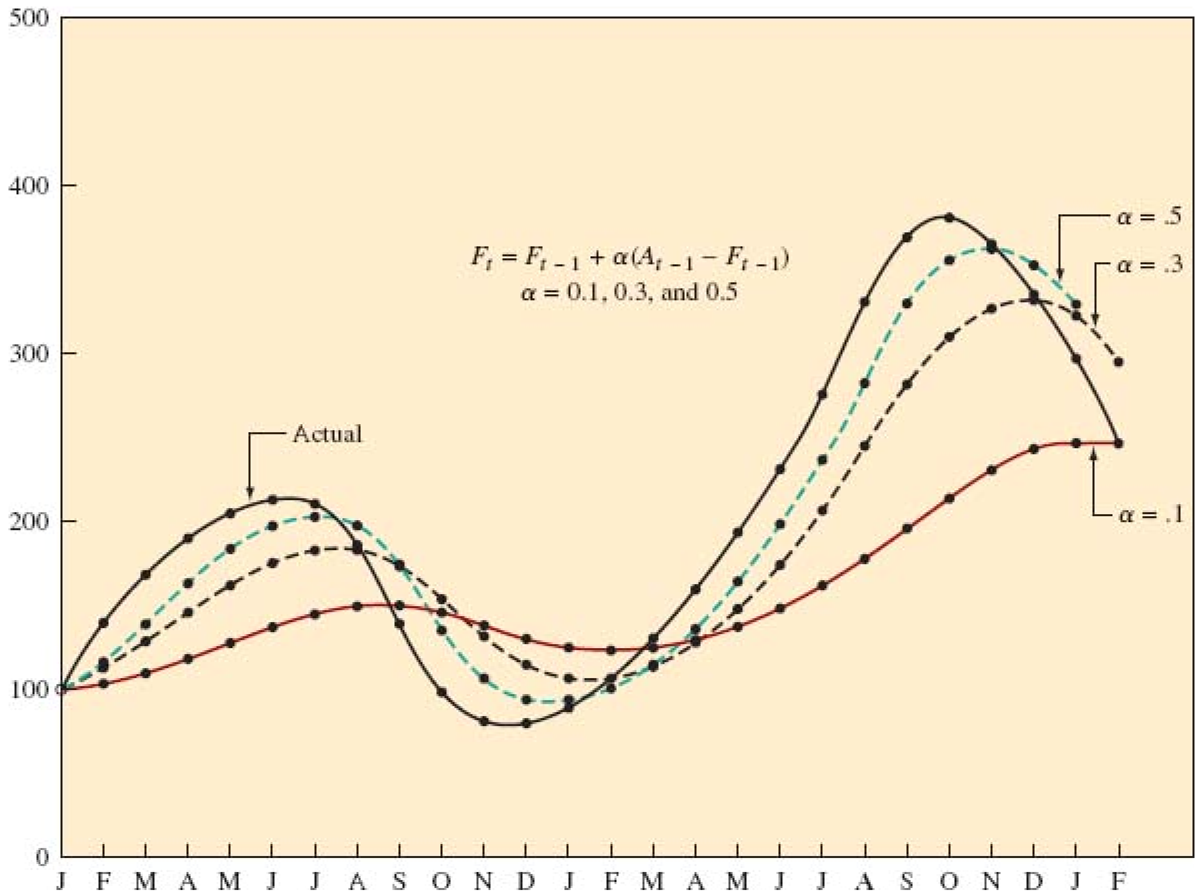
$$\begin{aligned} F_t &= F_{t-1} + (A_{t-1} - F_{t-1}) \\ &= 1{,}050 + 0.05(1{,}000 - 1{,}050) \\ &= 1{,}050 + 0.05(-50) \\ &= 1{,}047.5 \text{ units} \end{aligned}$$

Because the smoothing coefficient is small, the reaction of the new forecast to an error of 50 units is to decrease the next month's forecast by only $2^1/2$ units.

When exponential smoothing is first used for an item, an initial forecast may be obtained by using a simple estimate, like the first period's demand, or by using an average of preceding periods, such as the average of the first two or three periods.

Single exponential smoothing has the shortcoming of lagging changes in demand. Exhibit 18.5 presents actual data plotted as a smooth curve to show the lagging effects of the exponential forecasts. The forecast lags during an increase or decrease, but overshoots when a change in direction occurs. Note that the higher the value of alpha, the more closely the forecast follows the actual. To more closely track actual demand, a trend factor may be added. Adjusting the value of alpha also helps. This is termed *adaptive forecasting.* Both trend effects and adaptive forecasting are briefly explained in following sections.

exhibit 18.5 Exponential Forecasts versus Actual Demand for Units of a Product over Time Showing the Forecast Lag



Exponential Smoothing with Trend Remember that an upward or downward trend in data collected over a sequence of time periods causes the exponential forecast to always lag behind (be above or below) the actual occurrence. Exponentially smoothed forecasts can be corrected somewhat by adding in a trend adjustment. To correct the trend, we need two smoothing constants. Besides the smoothing constant α, the trend equation also uses a **smoothing constant delta (δ)**. Both alpha and delta reduce the impact of the error that occurs between the actual and the forecast. If both alpha and delta are not included, the trend overreacts to errors.

Smoothing constant delta (δ)
An additional parameter used in an exponential smoothing equation that includes an adjustment for trend.

To get the trend equation going, the first time it is used the trend value must be entered manually. This initial trend value can be an educated guess or a computation based on observed past data.

The equations to compute the forecast including trend (FIT) are

$$F_t = \text{FIT}_{t-1} + \alpha(A_{t-1} - \text{FIT}_{t-1}) \quad \textbf{[18.4]}$$

$$T_t = \text{T}_{t-1} + \delta(F_t + \text{FIT}_{t-1}) \quad \textbf{[18.5]}$$

$$\text{FIT}_t = F_t + Tt \quad \textbf{[18.6]}$$

where

F_t = The exponentially smoothed forecast that does not include trend for period t

T_t = The exponentially smoothed trend for period t

FIT_t = The forecast including trend for period t

FIT_{t-1} = The forecast including trend made for the prior period

A_{t-1}	= The actual demand for the prior period
α	= Smoothing constant (alpha)
δ	= Smoothing constant (delta)

To make an exponential forecast that includes trend, step through the equations one at a time.

Step 1: Using equation 18.4 make a forecast that is not adjusted for trend. This uses the previous forecast and previous actual demand.

Step 2: Using equation 18.5 update the estimate of trend using the previous trend estimate, the unadjusted forecast just made, and the previous forecast.

Step 3: Make a new forecast that includes trend by using the results from steps 1 and 2.

EXAMPLE 18.1: Forecast Including Trend

Assume a previous forecast including trend of 110 units, a previous trend estimate of 10 units, an alpha of .20, and a delta of .30. If actual demand turned out to be 115 rather than the forecast 110, calculate the forecast for the next period.

SOLUTION

The actual A_{t-1} is given as 115. Therefore,

$$
\begin{aligned}
F_t &= \text{FIT}_{t-1} + \alpha(A_{t-1} - \text{FIT}_{t-1}) \\
&= 110 + .2(115 - 110) = 111.0 \\
T_t &= T_{t-1} + \delta(F_t - \text{FIT}_{t-1}) \\
&= 10 + .3(111 - 110) = 10.3 \\
\text{FIT}_t &= F_t + T_t = 111.0 + 10.3 = 121.3
\end{aligned}
$$

If, instead of 121.3, the actual turned out to be 120, the sequence would be repeated and the forecast for the next period would be

$$
\begin{aligned}
F_{t+1} &= 121.3 + .2(120 - 121.3) = 121.04 \\
T_{t+1} &= 10.3 + .3(121.04 - 121.3) = 10.22 \\
\text{FIT}_{t+1} &= 121.04 + 10.22 = 131.26
\end{aligned}
$$

Exponential smoothing requires that the smoothing constants be given a value between 0 and 1. Typically fairly small values are used for alpha and delta in the range of .1 to .3. The values depend on how much random variation there is in demand and how steady the trend factor is. Later in the chapter, error measures are discussed that can be helpful in picking appropriate values for these parameters.

Linear Regression Analysis *Regression* can be defined as a functional relationship between two or more correlated variables. It is used to predict one variable given the other. The relationship is usually developed from observed data. The data should be plotted first to see if they appear linear or if at least parts of the data are linear. *Linear regression* refers to the special class of regression where the relationship between variables forms a straight line.

The linear regression line is of the form $Y = a + bt$, where Y is the value of the dependent variable that we are solving for, a is the Y intercept, b is the slope, and t is an index for the time period.

Linear regression is useful for long-term forecasting of major occurrences and aggregate planning. For example, linear regression would be very useful to forecast demands for product families. Even though demand for individual products within a family may vary widely during a time period, demand for the total product family is surprisingly smooth.

The major restriction in using **linear regression forecasting** is, as the name implies, that past data and future projections are assumed to fall in about a straight line. Although this does limit its application sometimes, if we use a shorter period of time, linear regression analysis can still be used. For example, there may be short segments of the longer period that are approximately linear.

Linear regression forecasting
A forecasting technique that fits a straight line to past demand data.

Linear regression is used both for time series forecasting and for causal relationship forecasting. When the dependent variable (usually the vertical axis on a graph) changes as a result of time (plotted as the horizontal axis), it is time series analysis. If one variable changes because of the change in another variable, this is a causal relationship (such as the number of deaths from lung cancer increasing with the number of people who smoke).

We use the following example to demonstrate linear least squares regression analysis:

EXAMPLE 18.2: Least Squares Method

A firm's sales for a product line during the 12 quarters of the past three years were as follows:

QUARTER	SALES
1	600
2	1,550
3	1,500
4	1,500
5	2,400
6	3,100
7	2,600
8	2,900
9	3,800
10	4,500
11	4,000
12	4,900

For a step-by-step walkthrough of this example, visit www.mhhe.com/jacobs14e_sbs_ch18.

The firm wants to forecast each quarter of the fourth year—that is, quarters 13, 14, 15, and 16.

SOLUTION

The least squares equation for linear regression is

$$Y = a + bt \qquad \textbf{[18.7]}$$

where

Y = Dependent variable computed by the equation
y = The actual dependent variable data point (used below)
a = Y intercept
b = Slope of the line
t = Time period

The least squares method tries to fit the line to the data *that minimizes the sum of the squares of the vertical distance* between each data point and its corresponding point on the line. If a straight line is drawn through the general area of the points, the difference between the point and the line is $y - Y$. Exhibit 18.6 shows these differences. The sum of the squares of the differences between the plotted data points and the line points is

$$(y_1 - Y_1)^2 + (y_2 - Y_2)^2 + \ldots + (y_{12} - Y_{12})^2$$

The best line to use is the one that minimizes this total.

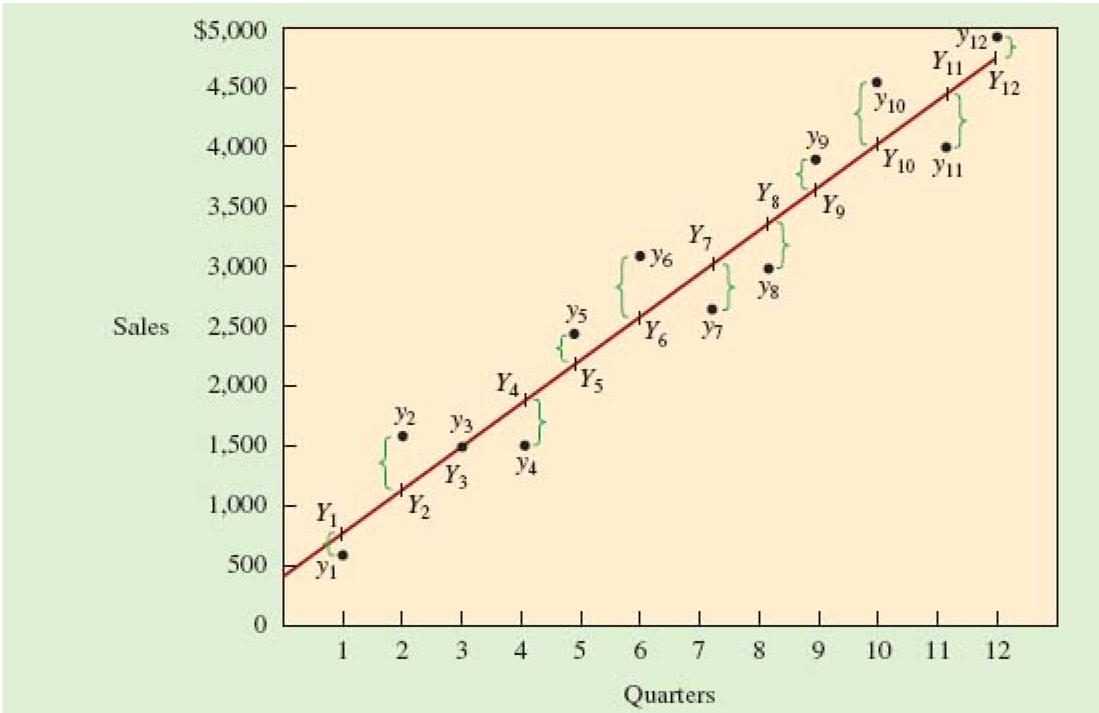
As before, the straight line equation is

$$Y = a + bt$$

In the least squares method, the equations for a and b are

$$b = \frac{\sum ty - n\bar{t}\cdot\bar{y}}{\sum t^2 - n\bar{t}^2} \qquad \textbf{[18.8]}$$

exhibit 18.6 Least Squares Regression Line



For the Excel template, visit www.mhhe.com/jacobs14e.

$$a = \bar{y} - b\bar{t} \tag{18.9}$$

where

- a = Y intercept
- b = Slope of the line
- $\bar{y}$ = Average of all ys
- $\bar{t}$ = Average of all ts
- t = t value at each data point
- y = y value at each data point
- n = Number of data points
- Y = Value of the dependent variable computed with the regression equation

Exhibit 18.7 shows these computations carried out for the 12 data points in the problem. Note that the final equation for Y shows an intercept of 441.67 and a slope of 359.6. The slope shows that for every unit change in t, Y changes by 359.6. Note that these calculations can be done with the INTERCEPT and SLOPE functions in Microsoft Excel.

Strictly based on the equation, forecasts for periods 13 through 16 would be

$$Y_{13} = 441.67 + 359.6(13) = 5{,}116.5$$
$$Y_{14} = 441.67 + 359.6(14) = 5{,}476.1$$
$$Y_{15} = 441.67 + 359.6(15) = 5{,}835.7$$
$$Y_{16} = 441.67 + 359.6(16) = 6{,}195.3$$

The standard error of estimate, or how well the line fits the data, is

$$S_{yt} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - Y_i)^2}{n-2}} \tag{18.10}$$

The standard error of estimate is computed from the second and last columns of Exhibit 18.7:

$$\begin{aligned} S_{yt} &= \sqrt{\frac{(600-801.3)^2+(1{,}550-1{,}160)^2+(1{,}500-1{,}520.5)^2+\cdots+(4{,}900-4{,}757.1)^2}{10}} \\ &= 363.9 \end{aligned}$$

exhibit 18.7 Least Squares Regression Analysis

(1) t	(2) y	(3) $t \times y$	(4) t^2	(5) y^2	(6) Y
1	600	600	1	360,000	801.3
2	1,550	3,100	4	2,402,500	1,160.9
3	1,500	4,500	9	2,250,000	1,520.5
4	1,500	6,000	16	2,250,000	1,880.1
5	2,400	12,000	25	5,760,000	2,239.7
6	3,100	18,600	36	9,610,000	2,599.4
7	2,600	18,200	49	6,760,000	2,959.0
8	2,900	23,200	64	8,410,000	3,318.6
9	3,800	34,200	81	14,440,000	3,678.2
10	4,500	45,000	100	20,250,000	4,037.8
11	4,000	44,000	121	16,000,000	4,397.4
12	4,900	58,800	144	24,010,000	4,757.1
78	33,350	268,200	650	112,502,500	

$\bar{t} = 6.5 \quad b = 359.6154$

$\bar{y} = 2{,}779.17 \quad a = 441.6667$

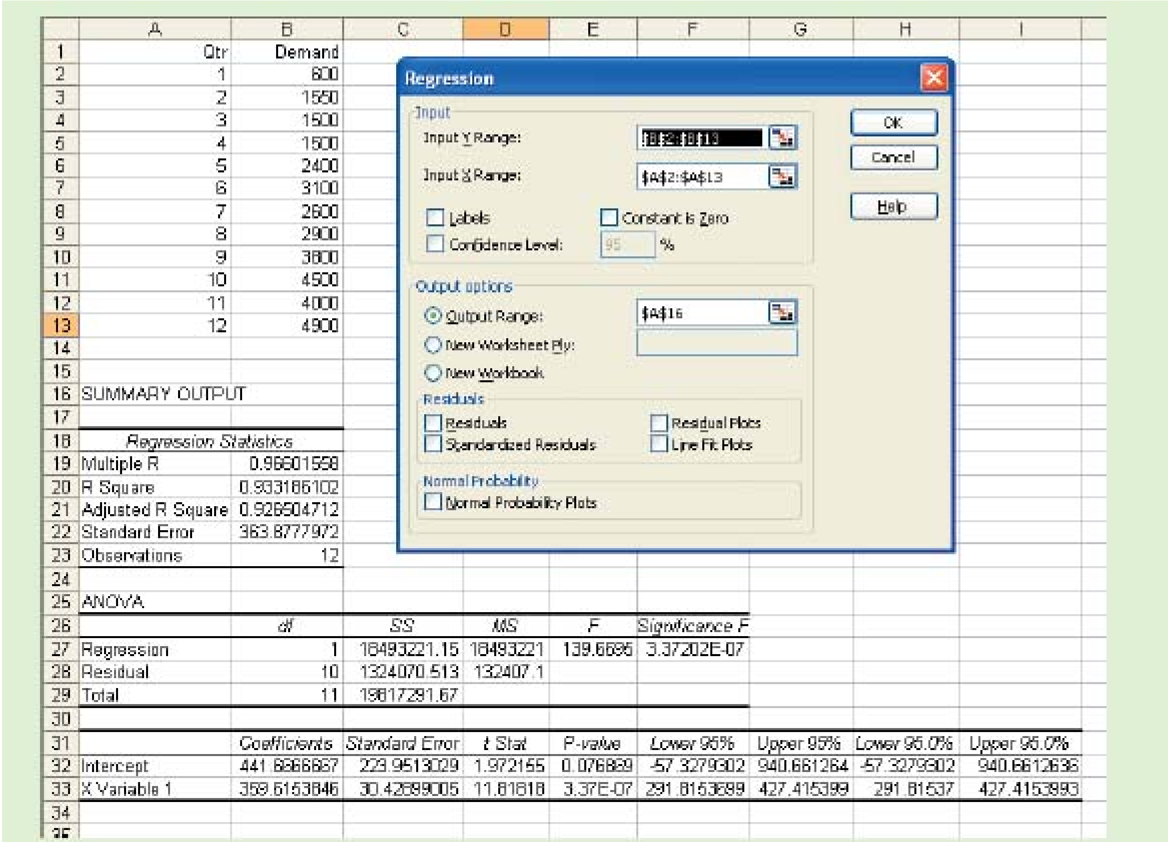
Therefore, $Y = 441.67 + 359.6t$

$S_{yt} = 363.9$

For the Excel template, visit www.mhhe.com/jacobs14e.

In addition to the INTERCEPT and SLOPE functions, Microsoft Excel has a very powerful regression tool designed to perform these calculations. (Note that it can also be used for moving average and exponential smoothing calculations.) To use the tool, a table is needed that contains data relevant to the problem (see Exhibit 18.8). The tool is part of the Data Analysis ToolPak that is accessed from the Data menu (you may need to add this to your Data options by using the Add-In option under File → Options → Add-Ins).

exhibit 18.8 Excel Regression Tool



For the Excel template, visit www.mhhe.com/jacobs14e.

To use the tool, first input the data in two columns in your spreadsheet, then access the Regression option from the → Data menu. Next, specify the Y Range, which is B2:B13, and the time periods in the X Range, which is A2:A13 in our example. Finally, an Output Range is specified. This is where you would like the results of the regression analysis placed in your spreadsheet. In the example, A16 is entered. There is some information provided that goes beyond what we have covered, but what you are looking for is the Intercept and X Variable coefficients that correspond to the intercept and slope values in the linear equation. These are in cells B32 and B33 in Exhibit 18.8.

We discuss the possible existence of seasonal components in the next section on decomposition of a time series.

Decomposition of a Time Series A *time series* can be defined as chronologically ordered data that may contain one or more components of demand: trend, seasonal, cyclical, autocorrelation, and random. **Decomposition** of a time series means identifying and separating the time series data into these components. In practice, it is relatively easy to identify the trend (even without mathematical analysis, it is usually easy to plot and see the direction of movement) and the seasonal component (by comparing the same period year to year). It is considerably more difficult to identify the cycles (these may be many months or years long), the autocorrelation, and the random components. (The forecaster usually calls random anything left over that cannot be identified as another component.)

Decomposition
The process of identifying and separating time series data into fundamental components such as trend and seasonality.

When demand contains both seasonal and trend effects at the same time, the question is how they relate to each other. In this description, we examine two types of seasonal variation: *additive* and *multiplicative*.

Additive seasonal variation simply assumes that the seasonal amount is a constant no matter what the trend or average amount is.

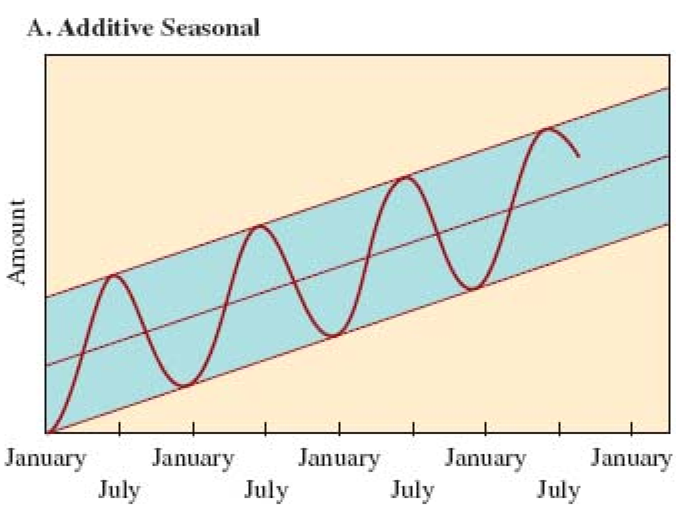
$$\text{Forecast including trend and seasonal} = \text{Trend} + \text{Seasonal}$$

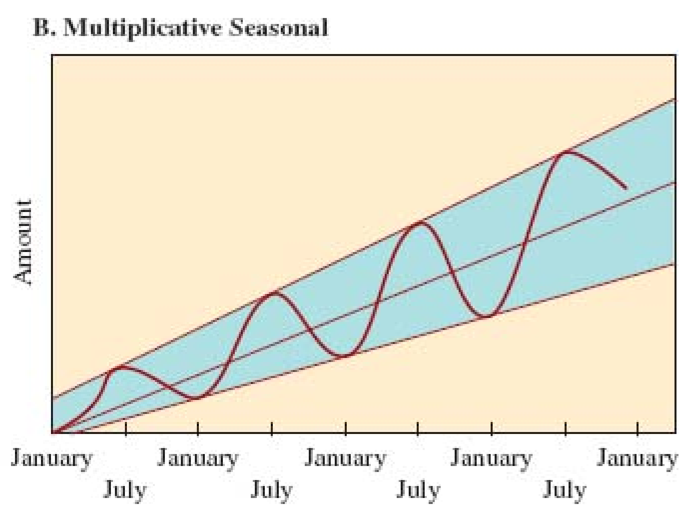
Exhibit 18.9A shows an example of increasing trend with constant seasonal amounts.

In multiplicative seasonal variation, the trend is multiplied by the seasonal factors.

$$\text{Forecast including trend and seasonal} = \text{Trend } 3 \text{ Seasonal factor}$$

exhibit 18.9 Additive and Multiplicative Seasonal Variation Superimposed on Changing Trend





For the Excel template, visit www.mhhe.com/jacobs14e.

Companies such as Toro manufacture lawnmowers and snow blowers to match seasonal demand. Using the same equipment and assembly lines provides better capacity utilization, workforce stability, productivity, and revenue.

Exhibit 18.9B shows the seasonal variation increasing as the trend increases because its size depends on the trend.

The multiplicative seasonal variation is the usual experience. Essentially, this says that the larger the basic amount projected, the larger the variation around this that we can expect.

A seasonal factor is the amount of correction needed in a time series to adjust for the season of the year.

We usually associate *seasonal* with a period of the year characterized by some particular activity. We use the word *cyclical* to indicate other than annual recurrent periods of repetitive activity.

The following examples show how seasonal indexes are determined and used to forecast (1) a simple calculation based on past seasonal data and (2) the trend and seasonal index from a hand-fit regression line. We follow this with a more formal procedure for the decomposition of data and forecasting using least squares regression.

EXAMPLE 18.3: Simple Proportion

Assume that in past years, a firm sold an average of 1,000 units of a particular product line each year. On the average, 200 units were sold in the spring, 350 in the summer, 300 in the fall, and 150 in the winter. The seasonal factor (or index) is the ratio of the amount sold during each season divided by the average for all seasons.

For a step-by-step walkthrough of this example, visit www.mhhe.com/jacobs14e_sbs_ch18.

SOLUTION

In this example, the yearly amount divided equally over all seasons is 1,000 4 4 = 250. The seasonal factors therefore are

	Past Sales	Average Sales for Each Season (1,000/4)	Seasonal Factor
Spring	200	250	200/250 = 0.8
Summer	350	250	350/250 = 1.4
Fall	300	250	300/250 = 1.2
Winter	150	250	150/250 = 0.6
Total	1,000		

Using these factors, if we expected demand for next year to be 1,100 units, we would forecast the demand to occur as

	Expected Demand for Next Year	Average Sales for Each Season (1,100/4)		Seasonal Factor		Next Year's Seasonal Forecast
Spring		275	×	0.8	=	220
Summer		275	×	1.4	=	385
Fall		275	×	1.2	=	330
Winter		275	×	0.6	=	165
Total	1,100					

The seasonal factor may be periodically updated as new data are available. The following example shows the seasonal factor and multiplicative seasonal variation.

EXAMPLE 18.4: Computing Trend and Seasonal Factor from a Linear Regression Line Obtained with Excel

Forecast the demand for each quarter of the next year using trend and seasonal factors. Demand for the past two years is in the following table:

Quarter	Amount
1	300
2	200
3	220
4	530
5	520
6	420
7	400
8	700

For a step-by-step walkthrough of this example, visit www.mhhe.com/jacobs14e_sbs_ch18.

SOLUTION

First, we plot as in Exhibit 18.10 and then calculate the slope and intercept using Excel. For Excel the quarters are numbered + through 8. The "known *ys*" are the amounts (300, 200, 220, etc.), and the "known *xs*" are the quarter numbers (1, 2, 3, etc.). We obtain a slope = 52.3 (rounded), and intercept = 176.1 (rounded). The equation for the line is

$$\text{Forecast Including Trend (FIT)} = 176.1 + 52.3t$$

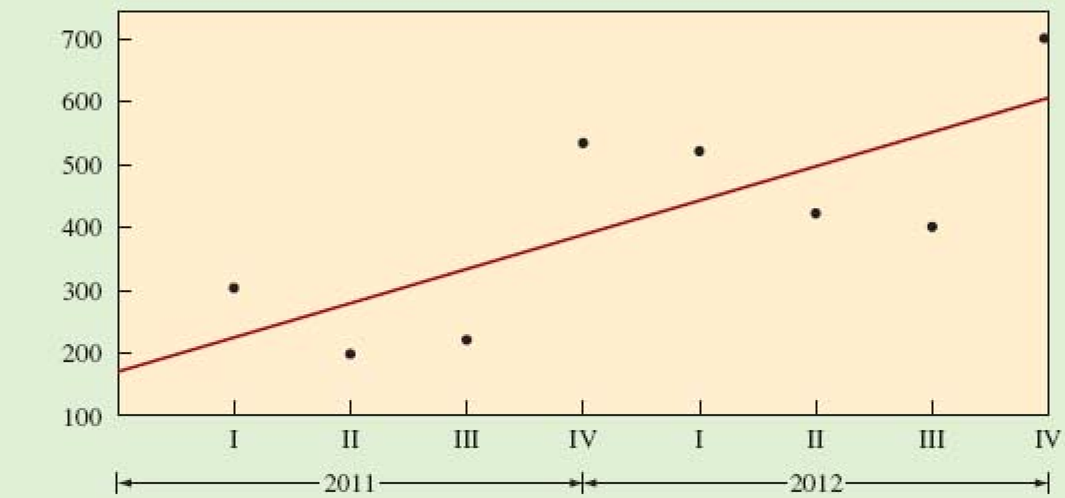
Next we can derive a seasonal index by comparing the actual data with the trend line, as in Exhibit 18.11. The seasonal factor was developed by averaging the same quarters in each year.

We can compute the 2013 forecast including trend and seasonal factors (FITS) as follows:

$$\text{FITS}_t = \text{FIT} \times \text{Seasonal}$$
$$\text{I—2013 FITS}_9 = [176.1 + 52.3(9)]1.25 = 808$$
$$\text{II—2013 FITS}_{10} = [176.1 + 52.3(10)]0.79 = 552$$
$$\text{III—2013 FITS}_{11} = [176.1 + 52.3(11)]0.70 = 526$$
$$\text{IV—2013 FITS}_{12} = [176.1 + 52.3(12)]1.28 = 1{,}029$$

Note, these numbers were calculated using Excel, so your numbers may differ slightly due to rounding.

exhibit 18.10 Computing a Seasonal Factor from the Actual Data and Trend Line



Quarter	Actual Amount	From Trend Equation $FIT_t = 176.1 + 52.3t$	Ratio of Actual ÷ Trend	Seasonal Factor (Average of Same Quarters in Both Years)
2011				
I	300	228.3	1.31	I 1.25
II	200	280.6	0.71	II 0.79
III	220	332.9	0.66	III 0.70
IV	530	385.1	1.38	IV 1.28
2012		—	—	
I	520	437.4	1.19	
II	420	489.6	0.86	
III	400	541.9	0.74	
IV	700	594.2	1.18	

excel

For the Excel template, visit www.mhhe.com/jacobs14e.

exhibit 18.11 Deseasonalized Demand

(1) Period (t)	(2) Quarter	(3) Actual Demand (y)	(4) Average of the Same Quarters of Each Year	(5) Seasonal Factor	(6) Deseasonalized Demand (y_d) Col. (3) ÷ Col. (5)	(7) t^2 (Col. 1)^2	(8) $t \times y_d$ Col. (1) × Col. (6)
1	I	600	(600 + 2,400 + 3,800)/3 = 2,266.7	0.82	735.7	1	735.7
2	II	1,550	(1,550 + 3,100 + 4,500)/3 = 3,050	1.10	1,412.4	4	2,824.7
3	III	1,500	(1,500 + 2,600 + 4,000)/3 = 2,700	0.97	1,544.0	9	4,631.9
4	IV	1,500	(1,500 + 2,900 + 4,900)/3 = 3,100	1.12	1,344.8	16	5,379.0
5	I	2,400		0.82	2,942.6	25	14,713.2
6	II	3,100		1.10	2,824.7	36	16,948.4
7	III	2,600		0.97	2,676.2	49	18,733.6
8	IV	2,900		1.12	2,599.9	64	20,798.9
9	I	3,800		0.82	4,659.2	81	41,932.7
10	II	4,500		1.10	4,100.4	100	41,004.1
11	III	4,000		0.97	4,117.3	121	45,290.1
12	IV	4,900		1.12	4,392.9	144	52,714.5
78		33,350*		12.03	33,350.1*	650	265,706.9

$\bar{t} = \frac{78}{12} = 6.5 \qquad b = \frac{\sum ty_d - n\bar{t}\bar{y}_d}{\sum t^2 - n\bar{t}^2} = \frac{265{,}706.9 - 12(6.5)2{,}779.2}{650 - 12(6.5)^2} = 342.2$

$\bar{y}_d = 33{,}350/12 = 2{,}779.2 \qquad a = \bar{y}_d - b\bar{t} = 2{,}779.2 - 342.2(6.5) = 554.9$

Therefore, $Y = a + bt = 554.9 + 342.2t$

***Column 3 and column 6 totals should be equal at 33,350. Differences are due to rounding. Column = was rounded to two decimal places.**

Decomposition Using Least Squares Regression This procedure is different from the previous one and in some cases may give better results. The approach described in Example 18.3 started by fitting a regression line, and given the line, the seasonal indexes are calculated. With this approach we start by calculating seasonal indexes; then using data that have been "deseasonalized" we estimate a trend line using linear regression. More formally, the process is:

1. Decompose the time series into its components.
   a. Find seasonal component.
   b. Deseasonalize the demand.
   c. Find trend component.
2. Forecast future values of each component.
   a. Project trend component into the future.
   b. Multiply trend component by seasonal component.

Exhibit 18.11 shows the decomposition of a time series using least squares regression and the same basic data we used in our first regression example. Each data point corresponds to using a single three-month quarter of the three-year (12-quarter) period. Our objective is to forecast demand for the four quarters of the fourth year.

Step 1. Determine the seasonal factor (or index). Exhibit 18.11 summarizes the calculations needed. Column 4 develops an average for the same quarters in the three-year period. For example, the first quarters of the three years are added together and divided by three. A seasonal factor is then derived by dividing that average by the general average for all 12 quarters $\left(\frac{33{,}350}{12}, \text{ or } 2{,}779\right)$. For example, this first quarter seasonal factor is $\frac{2{,}266.7}{2{,}779} = 0.82$. These are entered in column 5. Note that the seasonal factors are identical for similar quarters in each year.

Step 2. Deseasonalize the original data. To remove the seasonal effect on the data, we divide the original data by the seasonal factor. This step is called the deseasonalization of demand and is shown in column 6 of Exhibit 18.11.

Step 3. Develop a least squares regression line for the deseasonalized data. The purpose here is to develop an equation for the trend line Y, which we then modify with the seasonal factor. The procedure is the same as we used before:

$$Y = a + bt$$

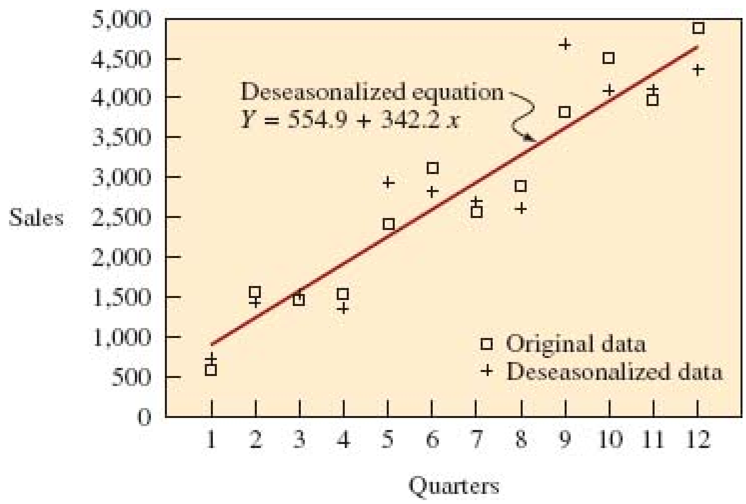
where

y_d = Deseasonalized demand (see Exhibit 18.11)
t = Quarter
Y = Demand computed using the regression equation
$Y = a + bt$
a = Y intercept
b = Slope of the line

The least squares calculations using columns 1, 7, and 8 of Exhibit 18.11 are shown in the lower section of the exhibit. The final deseasonalized equation for our data is $Y = 554.9 + 342.2t$. This straight line is shown in Exhibit 18.12.

Step 4. Project the regression line through the period to be forecast. Our purpose is to forecast periods 13 through 16. We start by solving the equation for Y at each of these periods (shown in step 5, column 3).

exhibit 18.12 Straight Line Graph of Deseasonalized Equation



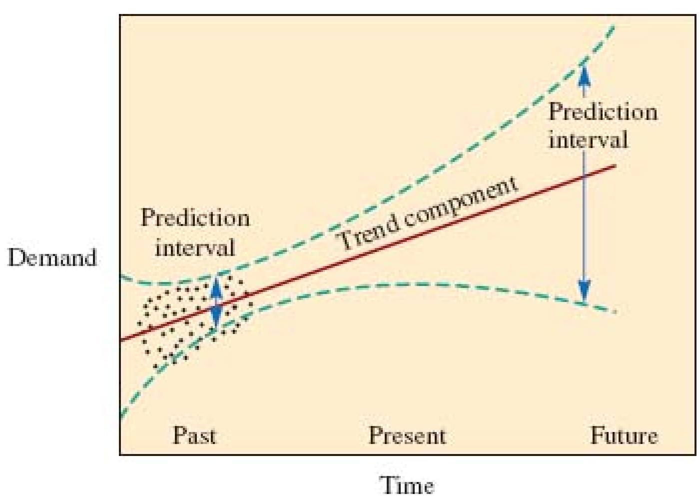
For the Excel template, visit www.mhhe.com/jacobs14e.

Step 5. Create the final forecast by adjusting the regression line by the seasonal factor. Recall that the *Y* equation has been deseasonalized. We now reverse the procedure by multiplying the quarterly data we derived by the seasonal factor for that quarter:

Period	Quarter	*Y* from Regression Line	Seasonal Factor	Forecast (*Y* × Seasonal Factor)
13	1	5,003.5	0.82	4,102.87
14	2	5,345.7	1.10	5,880.27
15	3	5,687.9	0.97	5,517.26
16	4	6,030.1	1.12	6,753.71

Our forecast is now complete.

exhibit 18.13 Prediction Intervals for Linear Trend



When a straight line is fitted through data points and then used for forecasting, errors can come from two sources. First, there are the usual errors similar to the standard deviation of any set of data. Second, there are errors that arise because the line is wrong. Exhibit 18.13 shows this error range. Instead of developing the statistics here, we will briefly show why the range broadens. First, visualize that one line is drawn that has some error such that it slants too steeply upward. Standard errors are then calculated for this line. Now visualize another line that slants too steeply downward. It also has a standard error. The total error range, for this analysis, consists of errors resulting from both lines as well as all other possible lines. We included this exhibit to show how the error range widens as we go further into the future.

Forecast Errors

In using the term **forecast error**, we are referring to the difference between what actually occurred and what was forecast. In statistics, these errors are called *residuals.* As long as the forecast value is within the confidence limits, as we discuss later under the heading, " Measurement of Error," this is not really an error since it is what we expected. But common usage refers to the difference as an error.

Forecast error

The difference between actual demand and what was forecast.

Demand for a product is generated through the interaction of a number of factors too complex to describe accurately in a model. Therefore, all forecasts certainly contain some error. In discussing forecast errors, it is convenient to distinguish between *sources of error* and the *measurement of error.*

Sources of Error Errors can come from a variety of sources. One common source that many forecasters are unaware of is projecting past trends into the future. For example, when we talk about statistical errors in regression analysis, we are referring to the deviations of observations from our regression line. It is common to attach a confidence band (that is, statistical control limits) to the regression line to reduce the unexplained error. But when we then use this regression line as a forecasting device by projecting it into the future, the error may not be correctly defined by the projected confidence band. This is because the confidence interval is based on past data; it may not hold for projected data points and therefore cannot be used with the same confidence. In fact, experience has shown that the actual errors tend to be greater than those predicted from forecast models.

Analytics

Errors can be classified as bias or random. *Bias errors* occur when a consistent mistake is made. Sources of bias include the failure to include the right variables; the use of the wrong relationships among variables; employing the wrong trend line; a mistaken shift in the seasonal demand from where it normally occurs; and the existence of some undetected secular trend. *Random errors* can be defined as those that cannot be explained by the forecast model being used.

Measurement of Error Several common terms used to describe the degree of error are *standard error, mean squared error* (or *variance*), and *mean absolute deviation.* In addition, tracking signals may be used to indicate any positive or negative bias in the forecast.

Standard error is discussed in the section on linear regression in this chapter. Because the standard error is the square root of a function, it is often more convenient to use the function itself. This is called the mean squared error, or variance.

The **mean absolute deviation (MAD)** was in vogue in the past but subsequently was ignored in favor of standard deviation and standard error measures. In recent years, MAD has made a comeback because of its simplicity and usefulness in obtaining tracking signals. MAD is the average error in the forecasts, using absolute values. It is valuable because MAD, like the standard deviation, measures the dispersion of some observed value from some expected value.

Mean absolute deviation (MAD)

The average of the absolute value of the actual forecast error.

MAD is computed using the differences between the actual demand and the forecast demand without regard to sign. It equals the sum of the absolute deviations divided by the number of data points or, stated in equation form,

$$\text{MAD} = \frac{\sum_{t=1}^{n} |\,\text{A}t - Ft\,|}{n} \qquad \textbf{[18.11]}$$

where

t = Period number

At = Actual demand for the period t

Ft = Forecast demand for the period *t*

n = Total number of periods

|| = A symbol used to indicate the absolute value disregarding positive and negative signs

When the errors that occur in the forecast are normally distributed (the usual case), the mean absolute deviation relates to the standard deviation as

$$1 \text{ standard deviation} = \sqrt{\frac{\pi}{2}} \times \text{MAD, or approximately } 1.25 \text{ MAD}$$

Conversely,

$$1 \text{ MAD is approximately } 0.8 \text{ standard deviation}$$

The standard deviation is the larger measure. If the MAD of a set of points was found to be 60 units, then the standard deviation would be approximately 75 units. In the usual statistical manner, if control limits were set at plus or minus 3 standard deviations (or 63.75 MADs), then 99.7 percent of the points would fall within these limits.

An additional measure of error that is often useful is the **mean absolute percent error (MAPE)**. This measure gauges the error relative to the average demand. For example, if the MAD is 10 units and average demand is 20 units, the error is large and significant, but relatively insignificant on an average demand of 1,000 units. MAPE is calculated by taking the MAD and dividing by the average demand,

Mean absolute percent error (MAPE)

The average error measured as a percentage of average demand.

$$\text{MAPE} = \frac{\text{MAD}}{\text{Average demand}} \qquad \textbf{[18.12]}$$

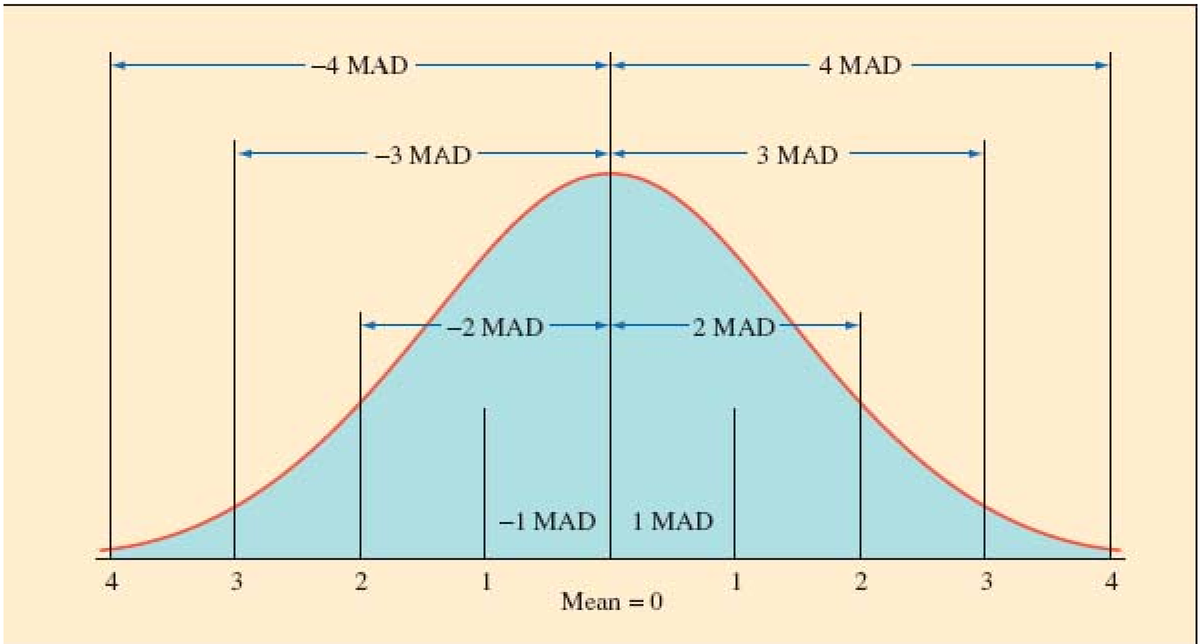
This is a useful measure because it is an estimate of how much error to expect with a forecast. So if the MAD were 10 and average demand 20, the MAPE would be 50 percent $\left(\frac{10}{20} = .50\right)$. In the case of an average demand of 1,000 units, the MAPE would be only 1 percent $\left(\frac{10}{1{,}000} = .01\right)$.

A **tracking signal** is a measurement that indicates whether the forecast average is keeping pace with any genuine upward or downward changes in demand. When a forecast is consistently low or high, it is referred to as a *biased* forecast. Exhibit 18.14 shows a normal distribution with a mean of 0 and a MAD equal to 1. Thus, if we compute the tracking signal and find

Tracking signal

A measure of whether the forecast is keeping pace with any genuine upward or downward changes in demand. This is used to detect forecast bias.

exhibit 18.14 A Normal Distribution with Mean = 0 and MAD = 1


–4 MAD
4 MAD
–3 MAD
3 MAD
–2 MAD
2 MAD
–1 MAD
1 MAD
4
3
2
1
1
2
3
4
Mean = 0

it equal to minus 2, we can see that the forecast model is providing forecasts that are quite a bit above the mean of the actual occurrences.

A tracking signal (TS) can be calculated using the arithmetic sum of forecast deviations divided by the mean absolute deviation:

$$TS = \frac{RSFE}{MAD} \quad [18.13]$$

where

RSFE = The running sum of forecast errors, considering the nature of the error. (For example, negative errors cancel positive errors and vice versa.)
MAD = The average of all the forecast errors (disregarding whether the deviations are positive or negative). It is the average of the absolute deviations.

Exhibit 18.15 illustrates the procedure for computing MAD and the tracking signal for a six-month period where the forecast had been set at a constant 1,000 and the actual demands that occurred are as shown. In this example, the forecast, on the average, was off by 66.7 units and the tracking signal was equal to 3.3 mean absolute deviations.

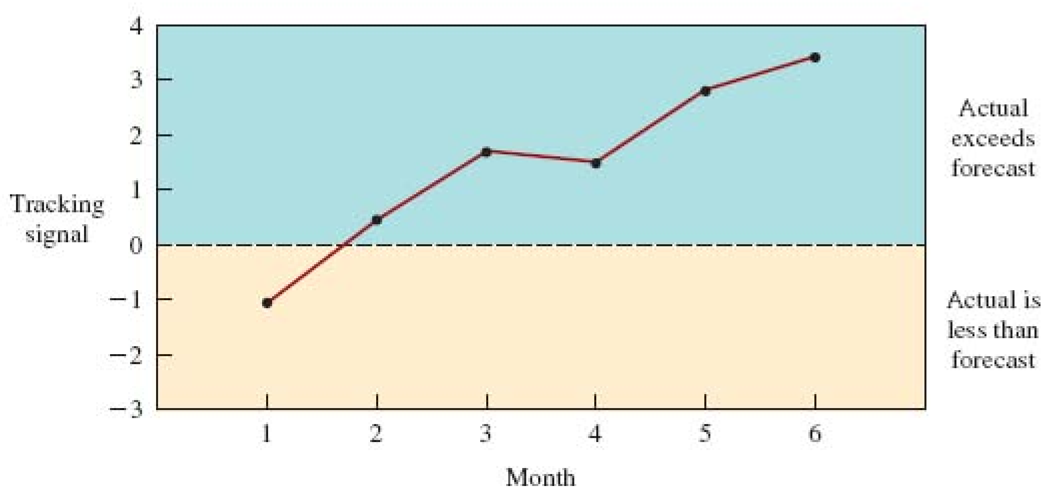
We can get a better feel for what the MAD and tracking signal mean by plotting the points on a graph. Though this is not completely legitimate from a sample-size standpoint, we plotted each month in Exhibit 18.15 to show the drift of the tracking signal. Note that it drifted from minus + MAD to plus 3.3 MADs. This happened because actual demand was greater than the forecast in four of the six periods. If the actual demand does not fall below the forecast to offset the continual positive RSFE, the tracking signal would continue to rise and we would conclude that assuming a demand of 1,000 is a bad forecast.

exhibit 18.15 Computing the Mean Absolute Deviation (MAD), the Running Sum of Forecast Errors (RSFE), and the Tracking Signal (TS) from Forecast and Actual Data

Month	Demand Forecast	Actual	Deviation	RSFE	Abs. Dev.	Sum of Abs. Dev.	MAD*	TS = RSFE† / MAD
1	1,000	950	−50	−50	50	50	50	−1
2	1,000	1,070	+70	+20	70	120	60	.33
3	1,000	1,100	+100	+120	100	220	73.3	1.64
4	1,000	960	−40	+80	40	260	65	1.2
5	1,000	1,090	+90	+170	90	350	70	2.4
6	1,000	1,050	+50	+220	50	400	66.7	3.3

*Overall, MAD = 400 ÷ 6 = 66.7. For all 6 months the average demand is 1,036.7. Given this, MAPE = 66.7/1,036.7 = 6.43%.

†Overall, $TS = \frac{RSFE}{MAD} = \frac{220}{66.7} = 3.3$ MADs.



For the Excel template, visit www.mhhe.com/jacobs14e.

Causal Relationship Forecasting

Causal relationship forecasting involves using independent variables other than time to predict future demand. To be of value for the purpose of forecasting, any independent variable must be a leading indicator. For example, we can expect that an extended period of rain will increase sales of umbrellas and raincoats. The rain causes the sale of rain gear. This is a causal relationship, where one occurrence causes another. If the causing element is known far enough in advance, it can be used as a basis for forecasting.

Causal relationship forecasting

Forecasting using independent variables other than time to predict future demand.

Often leading indicators are not causal relationships, but in some indirect way they may suggest that some other things might happen. Other noncausal relationships just seem to exist as a coincidence. The following shows one example of a forecast using a causal relationship.

Analytics

EXAMPLE 18.5: Forecasting Using a Causal Relationship

The Carpet City Store in Carpenteria has kept records of its sales (in square yards) each year, along with the number of permits for new houses in its area.

For a step-by-step walkthrough of this example, visit www.mhhe.com/jacobs14e_sbs_ch18.

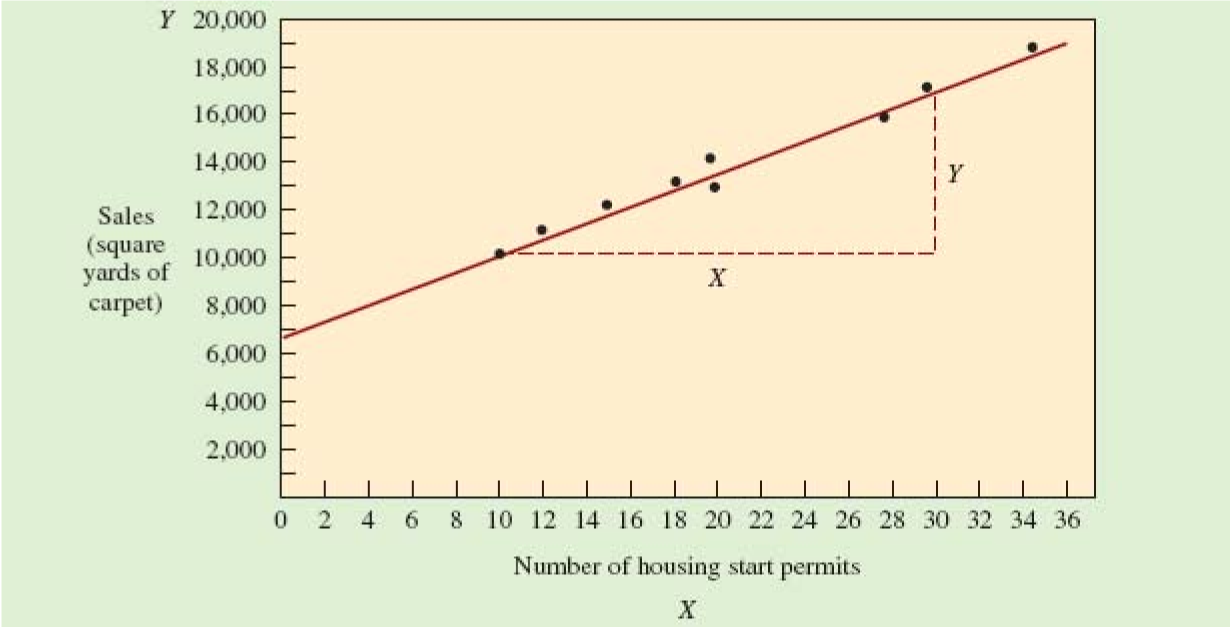
	Number of Housing Starts	
Year	Permits	Sales (in Sq. Yds.)
1	18	13,000
2	15	12,000
3	12	11,000
4	10	10,000
5	20	14,000
6	28	16,000
7	35	19,000
8	30	17,000
9	20	13,000

Carpet City's operations manager believes forecasting sales is possible if housing starts are known for that year. First, the data are plotted in Exhibit 18.16, with

x = Number of housing start permits

y = Sales of carpeting

exhibit 18.16 Causal Relationship: Sales to Housing Starts



For the Excel template, visit www.mhhe.com/jacobs14e.

Because the points appear to be in a straight line, the manager decides to use the linear relationship Y 5 $a + bx$.

SOLUTION

An easy way to solve this problem is to use the SLOPE and INTERCEPT functions in Excel. Given the data in the table, the SLOPE is equal to 344.2211 and the INTERCEPT is equal to 6698.492.

The manager interprets the slope as the average number of square yards of carpet sold for each new house built in the area. The forecasting equation is therefore

$$Y = 6698.492 + 344.2211x$$

Now suppose that there are 25 permits for houses to be built next year. The sales forecast would therefore be

$$6{,}698.492 + 344.2211(25) = 15{,}304.02 \text{ square yards}$$

In this problem, the lag between filing the permit with the appropriate agency and the new home owner coming to Carpet City to buy carpet makes a causal relationship feasible for forecasting.

Multiple Regression Analysis Another forecasting method is multiple regression analysis, in which a number of variables are considered, together with the effects of each on the item of interest. For example, in the home furnishings field, the effects of the number of marriages, housing starts, disposable income, and the trend can be expressed in a multiple regression equation as

$$S = A + B_m(M) + B_h(H) + B_i(I) + B_t(T)$$

where

S = Gross sales for year
A = Base sales, a starting point from which other factors have influence
M = Marriages during the year
H = Housing starts during the year
I = Annual disposable personal income
T = Time trend (first year = 1, second = 2, third = 3, and so forth)

B_m, B_h, B_i, and B_t represent the influence on expected sales of the numbers of marriages and housing starts, income, and trend.

Forecasting by multiple regression is appropriate when a number of factors influence a variable of interest—in this case, sales. Its difficulty lies with collecting all the additional data that is required to produce the forecast, especially data that comes from outside the firm. Fortunately, standard computer programs for multiple regression analysis are available, relieving the need for tedious manual calculation.

Microsoft Excel supports the time series analysis techniques described in this section. These functions are available under the Data Analysis tools for exponential smoothing, moving averages, and regression.

LO18–3

Apply qualitative techniques to forecast demand.

QUALITATIVE TECHNIQUES IN FORECASTING

Qualitative forecasting techniques generally take advantage of the knowledge of experts and require much judgment. These techniques typically involve processes that are well defined to those participating in the forecasting exercise. For example, in the case of forecasting the demand for new fashion merchandise in a retail store, the firm can include a combination of input from typical customers expressing preferences and from store managers who understand product mix and store volumes, where they view the merchandise and run through a series of exercises designed to bring the group to a consensus estimate. The point is that these are not wild guesses as to the expected demand, but rather involve a well-thought-out and structured decision-making approach.

These techniques are most useful when the product is new or there is little experience with selling into a new region. Here such information as knowledge of similar products, the habits of customers in the area, and how the product will be advertised and introduced may be important to estimate demand successfully. In some cases, it may even be useful to consider industry data and the experience of competing firms in making estimates of expected demand.

The following are samples of qualitative forecasting techniques.

Market Research

Firms often hire outside companies that specialize in *market research* to conduct this type of forecasting. You may have been involved in market surveys through a marketing class. Certainly you have not escaped telephone calls asking you about product preferences, your income, habits, and so on.

Market research is used mostly for product research in the sense of looking for new product ideas, likes and dislikes about existing products, which competitive products within a particular class are preferred, and so on. Again, the data collection methods are primarily surveys and interviews.

Panel Consensus

In a *panel consensus,* the idea that two heads are better than one is extrapolated to the idea that a panel of people from a variety of positions can develop a more reliable forecast than a narrower group. Panel forecasts are developed through open meetings with free exchange of ideas from all levels of management and individuals. The difficulty with this open style is that lower-level employees are intimidated by higher levels of management. For example, a salesperson in a particular product line may have a good estimate of future product demand but may not speak up to refute a much different estimate given by the vice president of marketing. The Delphi technique (which we discuss shortly) was developed to try to correct this impairment to free exchange.

When decisions in forecasting are at a broader, higher level (as when introducing a new product line or concerning strategic product decisions such as new marketing areas), the term *executive judgment* is generally used. The term is self-explanatory: a higher level of management is involved.

Historical Analogy

A Number of firms, including The Gilmore Research Group, now offer marketers software or databases to help them more accurately forecast sales for specific market areas, products, or segments.

In trying to forecast demand for a new product, an ideal situation would be one where an existing product or generic product could be used as a model. There are many ways to classify such analogies—for example, complementary products, substitutable or competitive products, and products as a function of income. Again, you have surely gotten a deluge of mail advertising products in a category similar to a product purchased via catalog, the Internet, or mail order. If you buy a DVD through the mail, you will receive more mail about new DVDs and DVD players. A causal relationship would be that demand for compact discs is caused by demand for DVD players. An analogy would be forecasting the demand for digital videodisc players by analyzing the historical demand for VCRs. The products are in the same general category of electronics and may be bought by consumers at similar rates. A simpler example would be toasters and coffeepots. A firm that already produces toasters and wants to produce coffeepots could use the toaster history as a likely growth model.

Delphi Method

As we mentioned under panel consensus, a statement or opinion of a higher-level person will likely be weighted more than that of a lower-level person. The worst case is where lower-level people feel threatened and do not contribute their true beliefs. To prevent this problem, the *Delphi method* conceals the identity of the individuals participating in the study. Everyone has the same weight. Procedurally, a moderator creates a questionnaire and distributes it to participants. Their responses are summed and given back to the entire group along with a new set of questions.

The step-by-step procedure for the Delphi method is:

1. Choose the experts to participate. There should be a variety of knowledgeable people in different areas.
2. Through a questionnaire (or e-mail), obtain forecasts (and any premises or qualifications for the forecasts) from all participants.
3. Summarize the results, and redistribute them to the participants along with appropriate new questions.
4. Summarize again, refining forecasts and conditions, and again develop new questions.
5. Repeat step 4 if necessary. Distribute the final results to all participants.

The Delphi technique can usually achieve satisfactory results in three rounds. The time required is a function of the number of participants, how much work is involved for them to develop their forecasts, and their speed in responding.

LO18–4

Apply collaborative techniques to forecast demand.

WEB-BASED FORECASTING: COLLABORATIVE PLANNING, FORECASTING, AND REPLENISHMENT (CPFR)

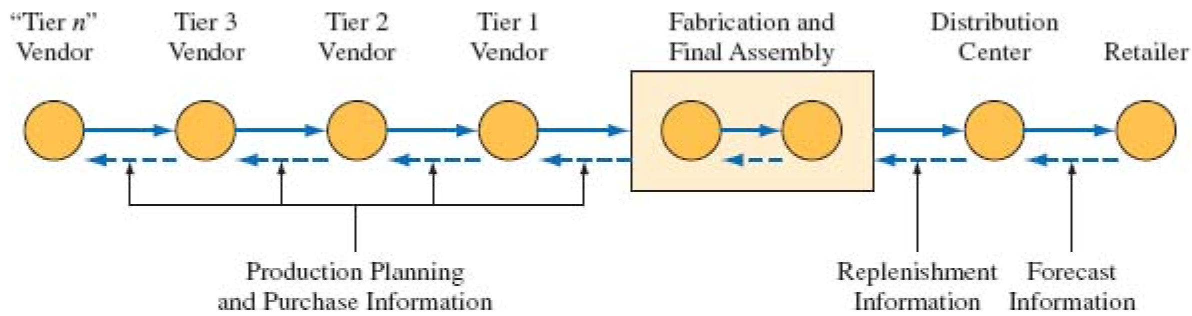
Collaborative Planning, Forecasting, and Replenishment (CPFR) is a web-based tool used to coordinate demand forecasting, production and purchase planning, and inventory replenishment between supply chain trading partners. CPFR is being used as a means of integrating all members of an *n*-tier supply chain, including manufacturers, distributors, and retailers. As depicted in Exhibit 18.17, the ideal point of collaboration utilizing CPFR is the retail-level demand forecast, which is successively used to synchronize forecasts, production, and replenishment plans upstream through the supply chain.

Collaborative Planning, Forecasting, and Replenishment (CPFR)
An Internet tool to coordinate forecasting, production, and purchasing in a firm's supply chain.

Although the methodology is applicable to any industry, CPFR applications to date have largely focused on the food, apparel, and general merchandise industries. The potential benefits of sharing information for enhanced planning visibility in any supply chain are enormous. Various estimates for cost savings attributable to improved supply chain coordination have been proposed, including $30 billion annually in the food industry alone.

CPFR's objective is to exchange selected internal information on a shared web server in order to provide for reliable, longer-term future views of demand in the supply chain. CPFR uses a cyclic and iterative approach to derive consensus supply chain forecasts. It consists of the following five steps:

Step 1. Creation of a front-end partnership agreement. This agreement specifies (1) objectives (e.g., inventory reductions, lost sales elimination, lower product obsolescence) to be gained through collaboration, (2) resource requirements (e.g., hardware, exhibit 18.17 *n*-Tier Supply Chain with Retail Activities



Note: Solid arrows represent material flows; dashed arrows represent information flows.

software, performance metrics) necessary for the collaboration, and (3) expectations of confidentiality concerning the prerequisite trust necessary to share sensitive company information, which represents a major implementation obstacle.

Step 2. Joint business planning. Typically partners create partnership strategies, design a joint calendar identifying the sequence and frequency of planning activities to follow that affect product flows, and specify exception criteria for handling planning variances between the trading partners' demand forecasts.

Process

Step 3. Development of demand forecasts. Forecast development may follow preexisting company procedures. Retailers should play a critical role as shared *point-of-sale* (POS) data permit the development of more accurate and timely expectations (compared with extrapolated warehouse withdrawals or aggregate store orders) for both retailers and vendors. Given the frequency of forecast generation and the potential for vast numbers of items requiring forecast preparation, a simple forecast procedure such as a moving average is commonly used within CPFR. Simple techniques are easily used in conjunction with expert knowledge of promotional or pricing events to modify forecast values accordingly.

Step 4. Sharing forecasts. Retailer (order forecasts) and vendor (sales forecasts) then electronically post their latest forecasts for a list of products on a shared server. The server examines pairs of corresponding forecasts and issues an exception notice for any forecast pair where the difference exceeds a preestablished safety margin (e.g., 5 percent). If the safety margin is exceeded, planners from both firms may collaborate electronically to derive a consensus forecast.

Step 5. Inventory replenishment. Once the corresponding forecasts are in agreement, the order forecast becomes an actual order, which commences the replenishment process. Each of these steps is then repeated iteratively in a continuous cycle, at varying times, by individual products and the calendar of events established between trading partners. For example, partners may review the front-end partnership agreement annually, evaluate the joint business plans quarterly, develop forecasts weekly to monthly, and replenish daily.

The early exchange of information between trading partners provides for reliable, longer-term future views of demand in the supply chain. The forward visibility based upon information sharing leads to a variety of benefits within supply chain partnerships.

As with most new corporate initiatives, there is skepticism and resistance to change. One of the largest hurdles hindering collaboration is the lack of trust over complete information sharing between supply chain partners. The conflicting objective between the profit-maximizing vendor and the cost-minimizing customer gives rise to adversarial supply chain relationships. Sharing sensitive operating data may enable one trading partner to take advantage of the other. Similarly, there is the potential loss of control as a barrier to implementation. Some companies are rightfully concerned about the idea of placing strategic data such as financial reports, manufacturing schedules, and inventory values online. Companies open themselves to security breaches. The front-end partnership agreements, nondisclosure agreements, and limited information access may help overcome these fears.

CONCEPT CONNECTIONS

LO18–1 Understand how forecasting is essential to supply chain planning.

Summary

Forecasts are essential to every business organization. It is important to consider the purpose of the forecast before selecting the technique. Strategic forecasts are typically longer term and usually involve forecasting demand for a group of products. Tactical forecasts would only cover a short period of time, at most a few weeks in the future, and would typically be for individual items. Forecasts are used in many different problems studied in this book.

Key Terms

Strategic forecasts

Tactical forecasts

LO18–2 Evaluate demand using quantitative forecasting models.

Summary

In this chapter, the focus is on time series analysis techniques. With a time series analysis, past demand data is used to predict future demand. Demand can be broken down or "decomposed" into basic elements, such as trend, seasonality, and random variation (there are other elements, but these are th e ones considered in this chapter). Four different time series models are evaluated: simple moving average, weighted moving average, exponential smoothing, and linear regression. Trend and seasonal components are analyzed for these problems. Causal relationship forecasting is different from time series (but commonly used) since it uses data other than past demand in making the forecast. The quality of a forecast is measured based on its error. Various measures exist including the average error, percentage of error, and bias. Bias occurs when a forecast is consistently higher or lower than actual demand.

Key Terms

Time series analysis

Moving average

Weighted moving average

Exponential smoothing

Smoothing constant alpha (α)

Smoothing constant delta (δ)

Linear regression forecasting

Decomposition

Forecast error

Mean absolute deviation (MAD)

Mean absolute percent error (MAPE)

Tracking signal

Causal relationship forecasting

Key Formulas

Simple moving average

[18.1] $$F_t = \frac{A_{t-1} + A_{t-2} + A_{t-3} + \cdots + A_{t-n}}{n}$$

Weighted moving average

[18.2] $$F_t = W_1 A_{t-1} + W_2 A_{t-2} \ldots + W_n A_{t-n}$$

Single exponential smoothing

[18.3] $F_t = F_{t-1} + (\alpha_{t-1} - F_{t-1})$

Exponential smoothing with trend

[18.4] $F_t = \text{FIT}_{t-1} + \alpha(A_{t-1} - \text{FIT}_{t-1})$

[18.5] $T_t = \text{T}_{t-1} + \delta(F_t + \text{FIT}_{t-1})$

[18.6] $\text{FIT}_t = F_t + Tt$

Least squares regression

[18.7] $Y = a + bt$

[18.8] $$b = \frac{\sum ty - n\bar{t} \cdot \bar{y}}{\sum t^2 - n\bar{t}^2}$$

[18.9] $$a = \bar{y} - b\bar{t}$$

Standard error of estimate

[18.10] $$S_{yt} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - Y_i)^2}{n-2}}$$

Mean absolute deviation

[18.11] $$\text{MAD} = \frac{\sum_{t=1}^{n} | \text{A}\mathrm{t} - F\mathrm{t} |}{n}$$

Mean absolute percent error

[18.12] $$\text{MAPE} = \frac{\text{MAD}}{\text{Average demand}}$$

Tracking signal

[18.13] $$\text{TS} = \frac{\text{RSFE}}{\text{MAD}}$$

LO18–3 Apply qualitative techniques to forecast demand.

Summary

Qualitative techniques depend more on judgment or the opinions of experts, and can be useful when past demand data are not available. These techniques typically involve a structured process so that experience can be acquired and accuracy assessed.

LO18–4 Apply collaborative techniques to forecast demand.

Summary

Collaboration between supply chain partners, such the manufacturer and the retailer selling a product, can be useful. Typically web-based technology is used to derive a forecast that is a consensus of all the participants. There are often great benefits to all participants due to the sharing of information and future planning visibility offered through the system.

Key Terms

Collaborative Planning, Forecasting, and Replenishment (CPFR)

Solved Problems

LO18–2 SOLVED PROBLEM 1

Sunrise Baking Company markets doughnuts through a chain of food stores. It has been experiencing over- and underproduction because of forecasting errors. The following data are its demand in dozens of doughnuts for the past four weeks. Doughnuts are made for the following day; for example, Sunday's doughnut production is for Monday's sales, Monday's production is for Tuesday's sales, and so forth. The bakery is closed Saturday, so Friday's production must satisfy demand for both Saturday and Sunday.

For the Excel template, visit www.mhhe.com/jacobs14e.

	4 Weeks Ago	3 Weeks Ago	2 Weeks Ago	Last Week
Monday	2,200	2,400	2,300	2,400
Tuesday	2,000	2,100	2,200	2,200
Wednesday	2,300	2,400	2,300	2,500
Thursday	1,800	1,900	1,800	2,000
Friday	1,900	1,800	2,100	2,000
Saturday	(closed on Saturday)			
Sunday	2,800	2,700	3,000	2,900

Make a forecast for this week on the following basis:

a. Daily, using a simple four-week moving average.

b. Daily, using a weighted moving average with weights of 0.40, 0.30, 0.20, and 0.10 (most recent to oldest week).

c. Sunrise is also planning its purchases of ingredients for bread production. If bread demand had been forecast for last week at 22,000 loaves and only 21,000 loaves were actually demanded, what would Sunrise's forecast be for this week using exponential smoothing with 5 0.10?

d. Suppose, with the forecast made in part (*c*), this week's demand actually turns out to be 22,500. What would the new forecast be for the next week?

SOLUTION

a. Simple moving average, four-week.

$$\begin{aligned}
\text{Monday}\ \frac{2{,}400+2{,}300+2{,}400+2{,}200}{4} &= \frac{9{,}300}{4} = 2{,}325\text{doz.}\\
\text{Tuesday} &= \frac{8{,}500}{4} = 2{,}125\text{doz.}\\
\text{Wednesday} &= \frac{9{,}500}{4} = 2{,}375\text{doz.}\\
\text{Thursday} &= \frac{7{,}500}{4} = 1{,}875\text{doz.}\\
\text{Friday} &= \frac{7{,}800}{4} = 1{,}950\text{doz.}\\
\text{Saturday and Sunday} &= \frac{11{,}400}{4} = 2{,}850\text{doz.}
\end{aligned}$$

b. Weighted average with weights of .40, .30, .20, and .10.

	(.10)		(.20)		(.30)		(.40)		
Monday	220	+	480	+	690	+	960	=	2,350
Tuesday	200	+	420	+	660	+	880	=	2,160
Wednesday	230	+	480	+	690	+	1,000	=	2,400
Thursday	180	+	380	+	540	+	800	=	1,900
Friday	190	+	360	+	630	+	800	=	1,980
Saturday and Sunday	280	+	540	+	900	+	1,160	=	2,880

c. Exponentially smoothed forecast for bread demand

$$\begin{aligned}
F_t &= F_{t-1} + \alpha(A_{t-1} - F_{t-1})\\
&= 22{,}000 + 0.10(21{,}000 - 22{,}000)\\
&= 22{,}000 - 100 = 21{,}900 \text{ loaves}
\end{aligned}$$

d. Exponentially smoothed forecast

$$\begin{aligned}
F_{t+1} &= 21{,}900 + .10(22{,}500 - 21{,}900)\\
&= 21{,}900 + .10(600) = 21{,}960 \text{ loaves}
\end{aligned}$$

LO18–2 SOLVED PROBLEM 2

Given the following information, make a forecast for May using exponential smoothing with trend and linear regression.

Month	January	February	March	April
Demand	700	760	780	790

For exponential smoothing with trend, assume that the previous forecast (for April) including trend (FIT) was 800 units, and the previous trend component (T) was 50 units. Also alpha (α) 5 .3 and delta (δ) = .1.

For linear regression, use the January through April demand data to fit the regression line. Use the Excel regression functions SLOPE and INTERCEPT to calculate these values.

SOLUTION

Exponential smoothing with trend

Use the following three steps to update the forecast each period:

1. Forecast without Trend $F_t = \text{FIT}_{t-1} + \alpha\,(A_{t-1} - \text{FIT}_{t-1})$
2. Update Trend estimate $T_t = T_{t-1} + \delta(F_t - \text{FIT}_{t-1})$

3. New Forecast including Trend $\text{FIT}_t = F_t + T_t$

$$\text{Given FIT}_{April} = 800$$
$$\text{and } T_{April} = 50$$
$$\text{Then } F_{May} = 800 + .3(790 - 800) = 797.$$
$$T_{May} = 50 + .1(797 - 800) = 49.7$$
$$\text{FIT}_{May} = 797 + 49.7 = 846.7$$

Linear regression

1. Set the problem up and calculate the slope and intercept as shown here.
2. Forecast using linear regression

$$F_t = a + bt$$

Where a is the “Slope” and b is the “Intercept” and t for May is = (the index for month 5).

$$\text{Then } F_{May} = 685 + 29(5) = 830$$

	A	B	C	D	E
1	Month	Demand			
2	1	700			
3	2	760			
4	3	780			
5	4	790			
6	Intercept	685	=INTERCEPT(B2:B5,A2:A5)		
7	Slope	29	=SLOPE(B2:B5,A2:A5)		
8					

SOLVED PROBLEM 3

Here are quarterly data for the past two years. From these data, prepare a forecast for the upcoming year using decomposition.

For the Excel template, visit www.mhhe.com/jacobs14e.

Period	Actual
1	300
2	540
3	885
4	580
5	416
6	760
7	1191
8	760

SOLUTION

(Note that the values you obtain may be slightly different due to rounding. The values given here were obtained using an Excel spreadsheet.)

(1) Period x	(2) Actual y	(3) Period Average	(4) Seasonal Factor	(5) Deseasonalized Demand y_d
1	300	358	0.527	568.99
2	540	650	0.957	564.09
3	885	1,038	1.529	578.92
4	580	670	0.987	587.79
5	416		0.527	789.01
6	760		0.957	793.91
7	1,191		1.529	779.08
8	760		0.987	770.21
Total	5,432	2,716	8.0	
Average	679	679	1	

Column 3 is seasonal average. For example, the first-quarter average is

$$\frac{300+416}{2} = 358$$

Column 4 is the quarter average (column 3) divided by the overall average (679). Column = is the actual data divided by the seasonal index. To determine t^2 and ty, we can construct a table as follows:

	Period t	Deseasonalized Demand (y_d)	t^2	ty_d
	1	568.99	1	569.0
	2	564.09	4	1128.2
	3	578.92	9	1736.7
	4	587.79	16	2351.2
	5	789.01	25	3945.0
	6	793.91	36	4763.4
	7	779.08	49	5453.6
	8	770.21	64	6161.7
Sums	36	5,432	204	26,108.8
Average	4.5	679		

Now we calculate regression results for deseasonalized data.

$$b = \frac{(26{,}108.8)-(8)(4.5)(679)}{(204)-(8)(4.5)^2} = 39.64$$

$$a = \bar{y}_d - b\bar{t}$$

$$a = 679 - 39.64\,(4.5) = 500.6$$

Therefore, the deseasonalized regression results are

$$Y = 500.6 + 39.64t$$

Period	Trend Forecast		Seasonal Factor		Final Forecast
9	857.4	×	0.527	=	452.0
10	897.0	×	0.957	=	858.7
11	936.7	×	1.529	=	1,431.9
12	976.3	×	0.987	=	963.4

SOLVED PROBLEM 4

A specific forecasting model was used to forecast demand for a product. The forecasts and the corresponding demand that subsequently occurred are shown below. Use MAD, tracking signal technique, and MAPE to evaluate the accuracy of the forecasting model.

For the Excel template, visit www.mhhe.com/jacobs14e.

	Actual	Forecast
October	700	660
November	760	840
December	780	750
January	790	835
February	850	910
March	950	890