Course Learning Outcomes for Unit III

Upon completion of this unit, students should be able to:

3. Apply time value of money techniques to various pricing (valuation) and budgeting problems.
   3.1 Explain compounding and discounting to calculate future and present values.
   3.2 Determine annuities from a single sum.
   3.3 Demonstrate how to apply time value of money concepts to complex cash streams.

Reading Assignment

Chapter 5: Time Value of Money - The Basics, pp. 126-150

Chapter 6: The Time Value of Money - Annuities and Other Topics, pp. 156-179

Unit Lesson

Creating value hinges on timing of cash flows. In finance, understanding time value of money is critical to value creation because cash flows resolve when an organization can create value. Discounting and compounding are key ingredients for looking at how time affects value creation. Take the case of Brite Phuteur, who has started a career and needs a new car.

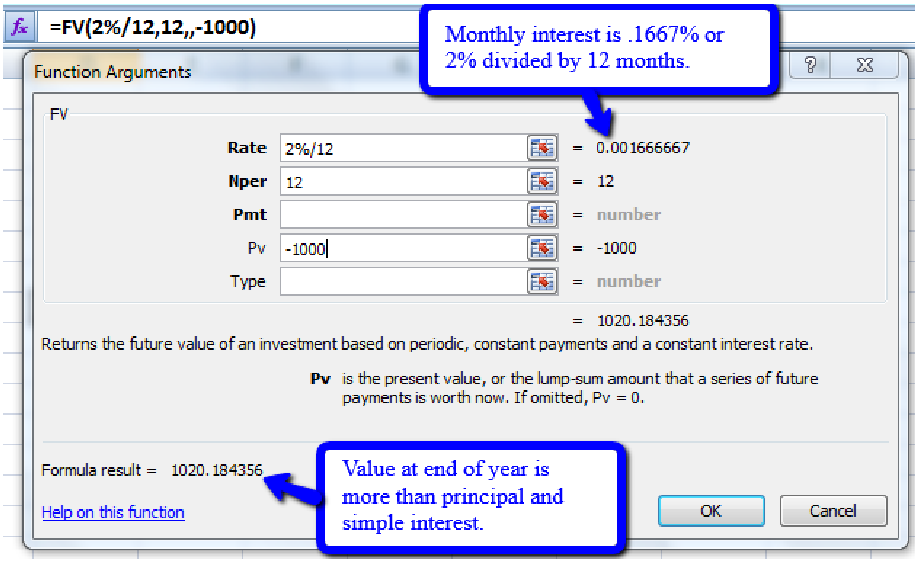
Phuteur has looked at different cars and has a good idea what he wants to buy. After looking at several dealerships, Phuteur noted pricing did not necessarily line up with his ideas about paying a fair price for the value he thought he would receive. Phuteur found dealerships varied their pricing on identical cars.

Despite not understanding price variances between dealers, Phuteur set aside money from his pay each pay period for a good down payment. Interest rates on savings at his local bank amounted to roughly 2%. If Phuteur set aside $100 a month at the end of the year, he wanted to know how much he would have after earning interest. This scenario is Phuteur's first experience where he needed to understand compounding. Phuteur asked his banker, Cash Kowntner, for help. Kowntner explained that compound interest adds interest earned to principal used when calculating future interest. For example, Phuteur started by putting $1,000 in savings in his bank, which pays 2% interest a year compounded monthly. Since Phuteur's local bank figures interest monthly, his monthly interest rate is just .16667% but compounds each of the 12 months during the year. Kowntner showed Phuteur a formula to calculate the amount he would have by year's end. A general future value formula solves this calculation as follows (Titman, Keown, & Martin, 2014, p. 130):

$FV = PV(1 + i)^n$, where FV equals future value, PV equals present value, i equals an interest rate, and n is the number of periods. Applying this equation, Phuteur can substitute his numbers into the formula as follows:

$FV = \$1,000(1 + .16667\%)^{12}$
Thus,
$FV = \$1,000(1.0016667)^{12}$
$FV = \$1,020.18$

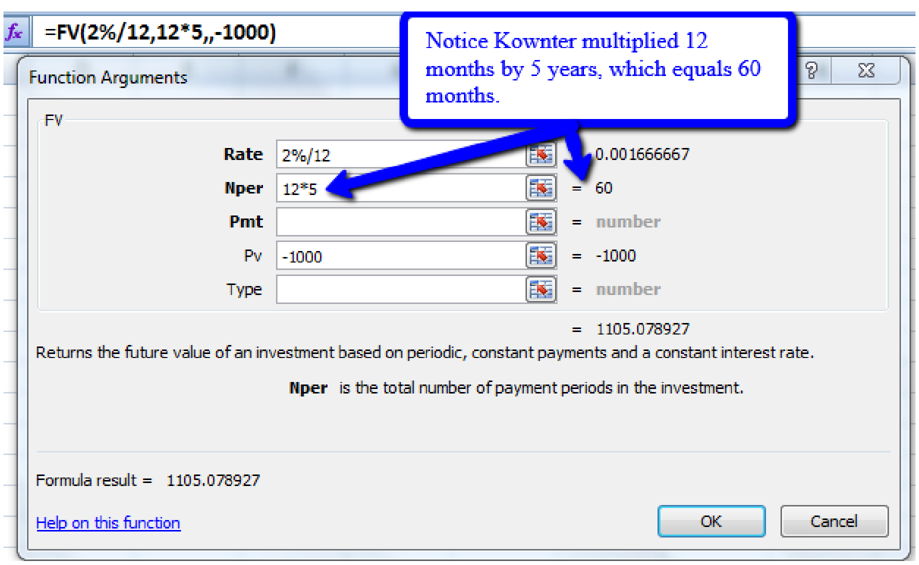
Phuteur observed he has more than simple interest, which simply adds 2% to $1,000, which only accounts for $1,020.00 of the $1,020.18 earned. Kownter showed Phuteur how to solve this problem in Excel as follows:



Despite compound interest exceeding simple interest, Phuteur noted the amount earned is not much more than he put in his savings to start. Phuteur asked Kowntner what he would earn if he left the money in the bank for five years. Kowntner changed his calculation as follows:

$FV = \$1{,}000(1 + .16667\%)^{12}$
Thus,
$FV = \$1{,}000(1.0016667)^{12*5}$
$FV = \$1{,}105.08$

Again Kowntner made the calculation in Excel as follows:



This time, Phuteur noted more interest earned by comparing $1,105.08 with $1,020.00 principal plus simple interest. Phuteur earned $85.08 in interest compared with just $.18 for just a year. Phuteur inferred the more periods the greater the interest earned. Kowntner explained compounding calculates interest on interest each period. Instead of simply computing interest on $1,000.00, compounding recalculates interest each period (in this case 60 periods), which grows interest earned.

Although compound interest showed a nice improvement in interest earned, Phuteur wondered what he could earn if he also saved $100 a month. Kownter told Phuteur, "Now you are talking about an annuity." Kowntner explained an annuity is a series of equal payments for a specific time. Again, Kowntner showed Phuteur how to calculate compound interest as follows:

$FV_n = PMT(1 + i)^{n-1} + PMT(1 + i)^{n-2} ... + PMT(1 + i)^1 + PMT(1 + i)^0$

Alternatively, one can apply a future value annuity factor (FVA) to an annuity payment (PMT) using the formula as follows:

$$FVA = FVA_n = \left[\frac{(1+i)^n - 1}{i}\right]$$

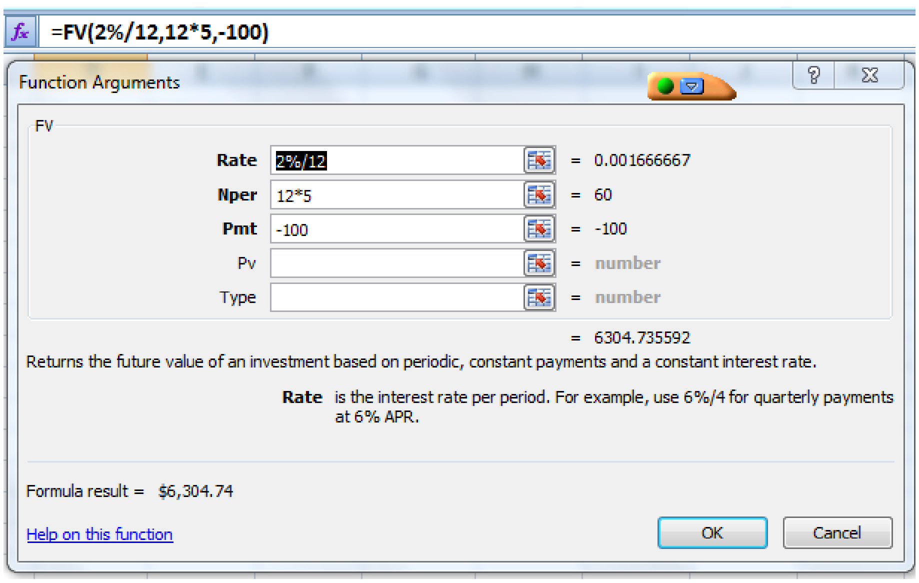
Applying this factor to the payment results in the following formula:

$$FV_n = PMT\ FV_n = PMT \left[\frac{(1+i)^n - 1}{i}\right]$$

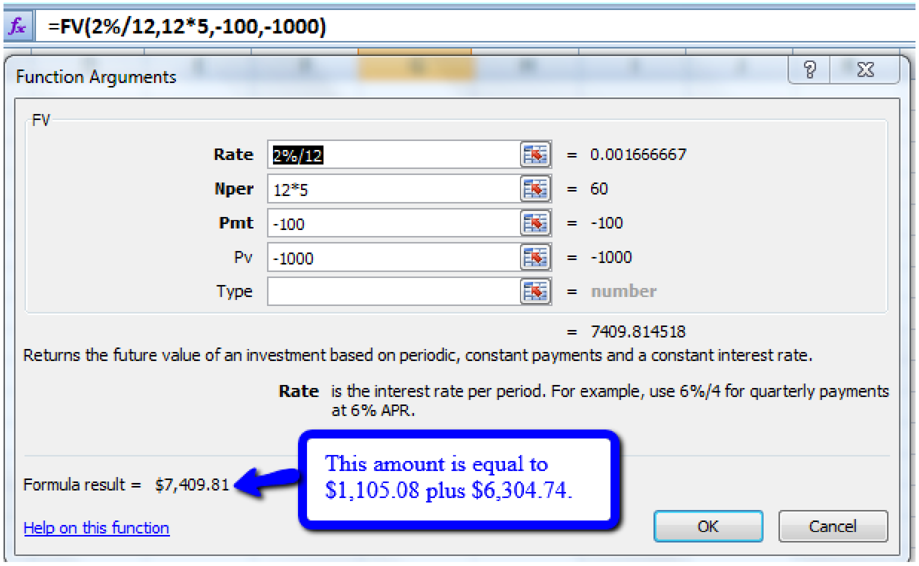
Using $100 a month savings deposit Kowntner filled in this formula as follows:

$FVA = [(1.001667)^{60}/.001667]$
$FVA = 63.04734$
$FV_{60} = 100(63.04734)$
$FV_{60} = 6304.74$

Alternatively, Kowntner again substitutes in Excel as follows:



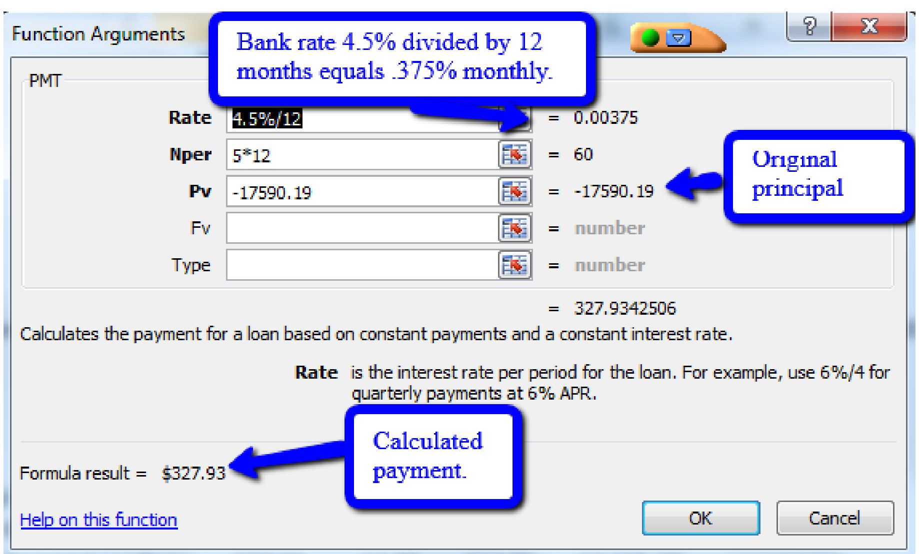
With both an initial deposit of $1,000 in savings and an added $100 a month, the result is as follows:



Phuteur felt better about saving for a down payment after calculating what he would have. Interest earned over five years is $409.82 ($105.08 plus $304.74) plus principal of $7,000 ($1,000 plus $6,000 or 60 payments times $100). Compound interest allowed Phuteur to grow his money by adding interest on interest earned.

Besides resolving he could have a large down payment, Phuteur still needed to know what he would have to pay monthly on a car. Phuteur looked at several cars on the Internet and settled on a car that sells for $25,000. Phuteur figured he would have a balance of $17,509.81 ($25,000 minus $7,409.81) he would need to finance. Phuteur asked at the bank what rate he would have to pay on a car loan. The bank gave Phuteur a rate of 4.5%. Phuteur asked Kowntar how he could calculate his payments. Kowntar responded by saying it might benefit Phuteur to prepare an amortization table to break down his payments between principal and interest.

Kowntar explained first Phuteur would need to calculate his payment. Kowntar showed Phuteur how to calculate his payment in Excel as follows:



Kownter assumed a normal five year loan and interest would compound monthly. Using a variant of the formula used to calculate the future value (FV) Kownter calculated Phuteur's payment as follows:

PV_n = PMT $PV = PMT\left[\frac{\frac{1}{(1+\frac{i}{m})^{n\times m}}}{i/m}\right]$

Substituting the above information Kownter solved the equation for the payment.

$\$17{,}509.81 = PMT\left[\frac{\frac{1}{(1+\frac{.045}{12})^{5\times 12}}}{.045/12}\right]$

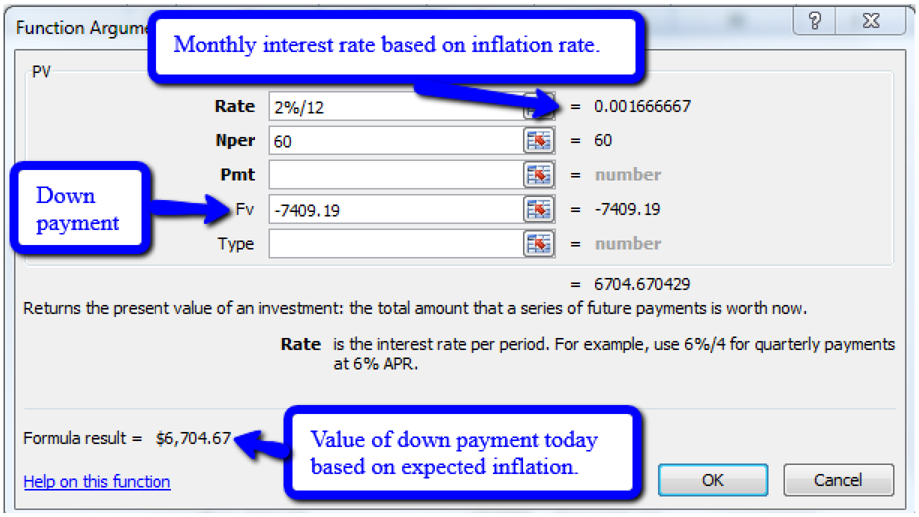
$17,509.81 = PMT(53.63938)
PMT = $17,590.81/$53.64197
PMT = 327.93

Kownter prepared the following amortization table in Excel:

		Payments	Interest	Principal Reduction	Principal
Original balance					$17,590.19
Payment	1	327.93	65.96	261.97	17,328.22
Payment	2	327.93	64.98	262.95	17,065.27
Payment	3	327.93	63.99	263.94	16,801.33
Payment	4	327.93	63.00	264.93	16,536.40
Payment	5	327.93	62.01	265.92	16,270.47
Payment	55	327.93	7.28	320.65	1,621.39
Payment	56	327.93	6.08	321.85	1,299.53
Payment	57	327.93	4.87	323.06	976.47
Payment	58	327.93	3.66	324.27	652.20
Payment	59	327.93	2.45	325.49	326.71
Payment	60	327.93	1.23	326.71	0.00
		19,676.06	2,085.87	17,590.19	

With Kownter's table, Phuteur had an idea what interest and principal he would have to pay each month. Payments for 60 months is an annuity because they are equal for a set time. Phuteur noticed early payments had higher interest as part of payments, but later payments included more principal.

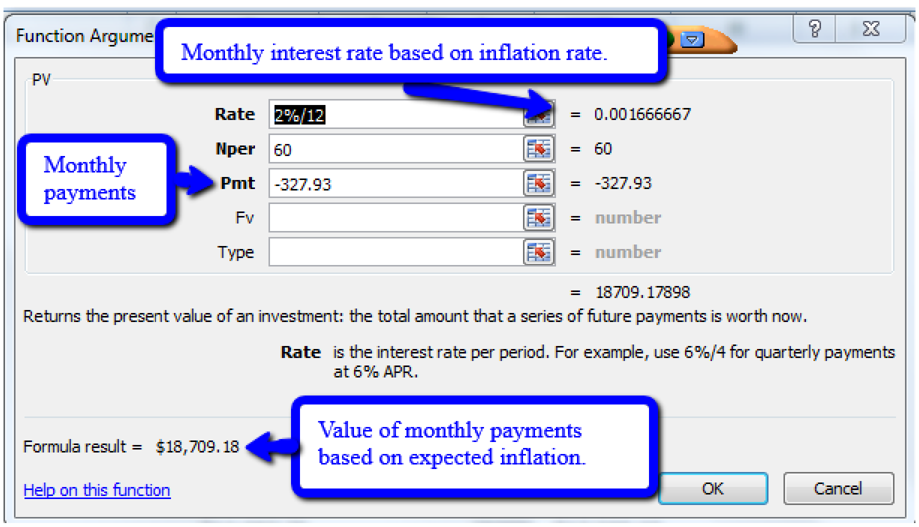
Because Kownter based payments on suggested retail price as suggested by the manufacturer, Phuteur wondered if he could buy his car for less. Phuteur began looking at current prices to see what different dealers offer. In today's prices, a similar car sold for $21,500. Kownter helped Phuteur explained to Kownter how discounting the payments based on inflation rate of 2% might give him a better idea of the value today. Kownter first discounted the $7,409.19 down payment to express it in today's dollars as follows:



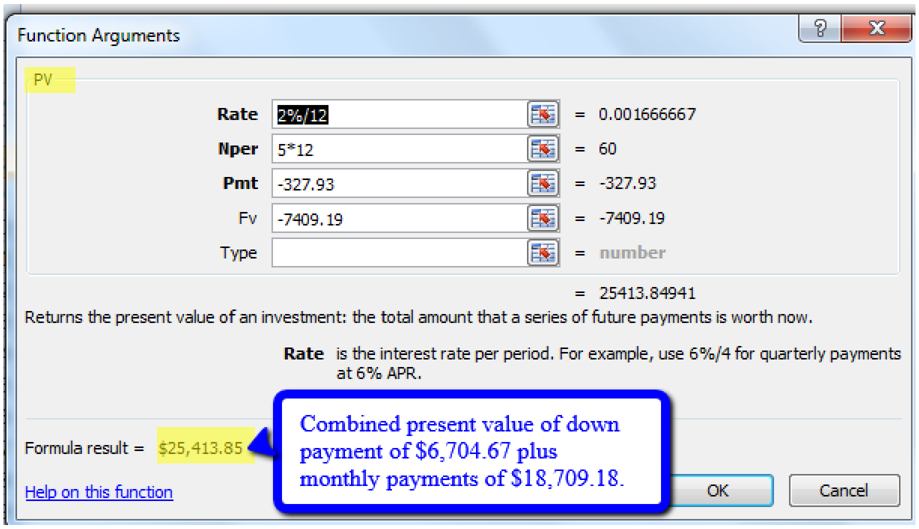
Kownter also solves for present value using the following formula:

PV = FVx1/(1 + i)^n or PV(1+i)$^{-n}$
In Phuteur's case substituting the values looks like the following:
PV = \$7,409.19 x 1/(1 + .00166667)^{60}
PV = \$7,409.19 x 1/(1.01004)
PV = \$7,409.19 x .90491
PV = \$6,704.67

Next, Kownter calculated the present value of the payments Phuteur would have to make as follows:



Combining the down payment and interest payments Kownter found the value today is as follows:



Comparing the $25,413.85 with what Phuteur projected five years from now it appears $25,000 is a fair price. Remember, however, Phuteur found one dealer selling the car he wanted for $21,500 at today's prices. This price suggests the $25,000 price in five years is too high, and Phuteur might well do better than the price Kownter used to calculate future payments. Assuming $25,000 is more than Phuteur would have to pay in five years, he likely will have lower payments based on dealer price variation. This revelation excited Phuteur about saving for his new car.

In summary, compounding cash flows results in future values, while discounting cash flows, brings future cash flows back to today's dollars for the sake of comparison. Some cash streams are complex and have both—a single sum plus an annuity, or series of equal payments for a set time. Phuteur benefited from doing his homework and reaching out to his friend at the bank. Phuteur did a good job with his homework and is now ready to buy a car when the time comes.

Reference

Titman, S., Keown, A. J., & Martin J. D. (2014). *Financial management: Principles and applications* (12th ed.). Upper Saddle River, NJ: Pearson.

Suggested Reading

In order to access the resource below, you must first log into the myCSU Student Portal and access the ABI/Inform database within the CSU Online Library.

This article further explores the concept of time value of money.

Benshoof, M. (2005). The time value of money. *Professional Builder, 70*(3), 74.

Learning Activities (Nongraded)

The following video tutorials will help you with the concepts covered in the textbook. You are strongly encouraged to watch these videos prior to starting the unit assessment.

Click here for CheckPoint 5.2 Calculating Future Value
Click here for CheckPoint 5.3 Calculating Future Values Using Non-Annual Compounding Periods
Click here for CheckPoint 5.4 Calculating Present Value
Click here for CheckPoint 5.5 Solving for n
Click here for CheckPoint 5.6 Solving for i
Click here for CheckPoint 5.7 Calculating EAR
Click here for Checkpoint 6.1 Solving for an Ordinary Annuity Payment
Click here for Checkpoint 6.2 Present Value an Ordinary Annuity
Click here for Checkpoint 6.3 Determining the Outstanding Balance on a Loan
Click here for Checkpoint 6.4 Calculation of a Level Perpetuity
Click here for Checkpoint 6.5 Calculation of a Growing Perpetuity
Click here for Checkpoint 6.6 Calculation of a Complex Cash Flow Stream

Nongraded Learning Activities are provided to aid students in their course of study. You do not have to submit them. If you have questions, contact your instructor for further guidance and information.